Instituto Superior Técnico

Digital Signal Processing (Processamento Digital de Sinais)

Test of 18/4/2016. Duration: 1.5 hours

Important notes:

- **Solve the problems in separate sheets.** You may solve several items of each problem in the same sheet.
- Identify all sheets with your student number and your first and last names.
- Present all answers in a clear, ordered and detailed manner.
- In calculations, keep at least three significant digits, and present all steps, with a brief explanation of each one.
- **Justify all answers and all calculation steps.**

Problem 1

Consider the discrete-time system defined by the equation $y(n) = x(n) - 2x(n-1) + 2x(n-3)$.

a) (*1.4 marks*) Is the system causal? And is it stable?

b) (*1.4 marks*) Find the system's impulse response.

Problem 2

Consider the causal discrete-time system that obeys the equation $y(n) - 0.3y(n-1) - 0.1y(n-2) = 2x(n)$.

a) (*1.8 marks*) Is the system stable?

b) (*1.8 marks*) Find the expression of the system's frequency response.

c) (*1.8 marks*) Find the system's response to the signal $x(n) = \sin(3n)$. Express that response as a real signal.

Problem 3

A continuous-time signal $x_c(t)$ with maximum frequency $\omega_M = 2000\pi$ is sampled with a sampling period of 0.2 ms, resulting in the discrete-time signal $x_d(n)$.

a) (*1.3 marks*) Can the signal x_c be exactly reconstructed from the signal x_d?

b) (*1.5 marks*) What continuous-time frequency (i.e., what frequency of the signal x_c) will the discrete-time frequency $\omega = \pi/5$, in $X_d(e^{j\omega})$, correspond to?

c) We create a finite-duration signal

$$x(n) = \begin{cases} x_d(n) & 0 \le n \le 99, \\ 0 & \text{other cases,} \end{cases}$$

and we compute its 100-point DFT, $X(k)$. What continuous-time frequencies will the following DFT samples correspond to?

i) (*1.5 marks*) $X(20)$.

ii) (*1.5 marks*) $X(90)$.

Alternative: Indicate what *discrete-time* (instead of continuous-time) frequencies those samples correspond to. *Marks for this alternative:* i) *1.0 marks*; ii) *0.5 marks.*

Problem 4

a) (*2 marks*) Compute the 4-point circular convolution of the signals $x(n) = 2\delta(n) - 3\delta(n-2)$ and $y(n) = 3\delta(n-2) - 2\delta(n-3)$.

b) (*2 marks*) Compute the 4-point DFT of the signal $x(n)$ of item a).

Problem 5 (*2 marks*)

This problem is intended to distinguish the students that deal best with the topics studied in our course. In this problem, you must provide a very detailed and very carefully justified response.

A discrete-time random process x is created in the following idealized manner: We start with $x(n) = 0, \forall n$. Then, for each value of n, we flip a well-balanced coin. If we get heads, we increase by 1 the values of $x(n-1)$, $x(n)$ and $x(n+1)$. If we get tails, we decrease those values by 1. Find the power spectrum of the process that results after doing this for all values of n.

Note: There is a rather simple way to solve this problem. A significant mark is given to the use of a simple solution, because it shows that the student manipulates the topic's concepts well enough to find that solution.