VELOCITY ESTIMATION OF FAST MOVING TARGETS USING UNDERSAMPLED SAR RAW-DATA

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ABSTRACT
The paper presents a new methodology to retrieve slant-range velocity estimates for moving targets which induce a Doppler-shift beyond the Nyquist limit determined by the Pulse Repetition Frequency (PRF). The proposed scheme takes advantage of the fact that the range velocity of a moving target induces a Doppler-shift in the azimuth spectra which depends linearly on the fast-time frequency. We present results that take real and simulated SAR data.

1. INTRODUCTION
It is well known that a moving target induces a Doppler-shift and a Doppler-spread on the returned signal in the slow-time frequency domain [1]. Most of the techniques proposed in recent literature take advantage of this knowledge to retrieve the moving target image and velocity parameters [2],[3],[4]. The azimuth velocity of a moving target is the responsible for the spread in the slow-time frequency domain whereas the range velocity induces the Doppler-shift. Given a PRF, the Doppler-shift is confined to,

\[-\frac{PRF}{2} \leq f_D \leq \frac{PRF}{2},\]  

where \(f_D = 2v_r/\lambda\) is azimuth Doppler-shift induced by a moving target with range velocity \(v_r\), when the carrier wavelength is \(\lambda\). If the signal is aliased (the induced Doppler-shift exceeds \(PRF/2\)) it has been generally accepted that the true moving target range velocity cannot be uniquely determined using a single antenna and a single pulse scheduling [5],[6]. The traditional solution to resolve such targets consists in increasing the PRF [5], or alternatively, in using a non-uniform PRF as proposed in [6]. PRF increasing leads to a decrease in the maximum unambiguous range swath, besides the huge memory requirements to store the received signal. The use of a non-uniform 

PRF needs a non-conventional pulse scheduling. Moreover, non-uniform sampling introduces higher complexity in image reconstruction. Using typical SAR mission parameters, a single sensor, and uniform pulse scheduling, we readily conclude that the maximum unambiguous range velocity is usually very small [7].

The approach herein proposed to estimate the range velocity of moving targets with velocities above the maximum imposed by the PRF is based on the knowledge that the Doppler-shift in the azimuth spectra depends linearly on the radar fast-time frequency; i.e., the Doppler-shift varies with \(k\) proportionally to the true target range velocity. In the two dimensional frequency domain, a moving target return will exhibit a slope which is not subject to PRF limitations. We will present a methodology to retrieve the linear dependence of the Doppler-shift in the azimuth dimension with the fast-time frequency, thus computing an unaliased estimate of the moving target range velocity.

The developed methodology is not intended to achieve high accuracy on the range velocity estimation. Rather, it is designed to retrieve the azimuth spectral support where the Doppler-shift belongs. This information is crucial to retrieve the range velocity with high accuracy (see, e.g., [2]).

2. UNAMBIGUOUS DOPPLER-SHIFT ESTIMATION
In [2] we have shown that the returned echo \(A(k_u, k)\) from a moving target takes, in the slow-time frequency domain \(k_u\), the shape of the antenna radiation pattern \(g\) in the cross-range direction,

\[A(k_u, k) \propto g \left(\frac{1}{2\nu}(k_u - 2k\mu)\right),\]  

where \(k = 2\pi/\lambda\) is the wavenumber corresponding to the fast-time frequency domain \(f\). The shape \(g\) becomes shifted proportionally to \(\mu\) and expanded by \(\nu\). Symbol \(\mu = v_r/V\)
Fig. 1. Returned signal from a moving target with relative range velocity $\mu$.

denotes the moving target relative range velocity with respect to the sensor velocity $V$, and $\nu = (1 + v_a/V)$, where $v_a$ is the target cross-range speed. Usually $k$ is regarded as a constant and equal to $k_0 = 2\pi/\lambda_0$ where $\lambda_0$ is the carrier wavelength. In this work we drop this assumption. If the transmitted pulse has bandwidth $B$, then $k$ is confined to

$$k_{\text{min}} = -\frac{\pi B}{c} + k_0 \leq k \leq k_0 + \frac{\pi B}{c} = k_{\text{max}},$$

(3)

where $c$ is the propagation speed. For a moving target with relative range velocity $\mu$, one may expect from equation (2), that the returned signal $S(k_{\text{u}}, k)$ will exhibit a slope of $2\mu$ along the $k$ axis, as illustrated in Fig. 1, in the absence of any other returns. We can readily conclude that

$$k_{\text{u,m,a}} - k_{\text{u,s,a}} = 2(k_{\text{max}} - k_{\text{min}})\mu,$$

(4)

where $k_{\text{u,m,a}}$ and $k_{\text{u,s,a}}$ are the measured Doppler-shifts at fast-time frequencies $k_{\text{max}}$ and $k_{\text{min}}$, respectively. In the absence of noise, $k_{\text{u,m,a}}$ and $k_{\text{u,s,a}}$ could be inferred using a simple centroid technique. In this situation the relative velocity would thus be estimated as

$$\hat{\mu} = \frac{k_{\text{u,m,a}} - k_{\text{u,s,a}}}{2(k_{\text{max}} - k_{\text{min}})},$$

(5)

and would not be restricted to the maximum value imposed by the PRF.

In real situations the returned signal from a moving target is superimposed on the clutter returns, making impossible the use of a simple spectral centroid estimator. A scheme based on estimation theory is not easy to derive because we do not have any information about the signal to use as reference. However, the following facts apply:

- Assumption that the number of static scatterers is large, none is predominant, and that they are uniformly distributed within a wavelength, then the correlation of the static ground returns, in the $(k_{\text{u}}, k)$ domain decays very quickly [8];

- The signal from a moving target for a fixed $k = k_0$, exhibit high correlation with $S(k_{\text{u}}, k)$ for $k = k_0 + \Delta k$. The correlation will have a maximum at $k_{\text{u}} = 2\Delta k\mu$ (shown in appendix).

From the previous statements, one can intuitively establish the following methodology to achieve an estimate of $\mu$:

- Estimate the location of the moving targets using one of the methodologies proposed in [2], [4] or [9].

- Process the SAR raw-data as if there were only static targets. The moving targets will appear smeared and defocused.

- For each moving target:

  - Digital spotlight the moving target image in the spatial domain and re-synthesize its signature back to the $(k_{\text{u}}, k)$ domain as described in [7];
  - Compute the correlation $R_{S S_{\text{u}}}(k_{\text{u}})$ between $S_0(k_{\text{u}}, k_0)$ and $S(k_{\text{u}}, k)$ for all the transmitted pulse bandwidth;
  - Perform a linear regression on the maximum values of $R_{S S_{\text{u}}}(k_{\text{u}})$ along $k$ axis to estimate $\mu$ and subsequently retrieve the true target range velocity.

### 3. SIMULATION RESULTS

In this section we show some results obtained using the proposed methodology. Figure 2 shows a real SAR image where a simulated moving target signature was superimposed. The moving target is composed of four reflectors. The total dimension is $2 \times 2m$, and the target is moving with range speed 6.3 times the maximum allowed by the PRF (the mission parameters are presented in Table 1). In Fig. 3a), the corresponding magnitude image is presented in the spatial frequency domain $(k_{\text{u}}, k)$. The returns from the static ground behave as noise. In the frequency interval $k_{\text{u}} \in [2,3]$ rad/m we can see part of the moving target signature, which is very weak compared with the static

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Carrier frequency</td>
<td>10GHz</td>
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<tr>
<td>Chirp bandwidth</td>
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<td>Altitude</td>
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<tr>
<td>Velocity</td>
<td>637Km/h</td>
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<td>Look angle</td>
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<td>Antenna radiation pattern</td>
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<tr>
<td>Oversampling factor</td>
<td>1.5</td>
</tr>
</tbody>
</table>

*Table 1. Mission parameters used in simulation.*
Fig. 2. Reconstructed SAR image which contains a moving target with range velocity 6.3 times the maximum imposed by the mission PRF.

ground (SNR=-4.7dB). By taking as reference the signal $S(k_u, k_0 = 209.4)$ and performing the correlation proposed in the previous section, we obtain the result illustrated in Fig. 3b). This figure clearly displays a maximum whose $k_u$ coordinate varies linearly along $k$ axis. Performing a linear regression on the pairs $(k, k_u)$ corresponding to the referred maxima we obtain $\hat{\mu} = 0.0463$ (the true value is $\mu = 0.0472$), which corresponds to an error of $0.57 km/h$ (the object is moving with range velocity of $30 km/h$).

Fig. 4 plots Monte Carlo results for an object with velocity which goes up to 15 times the maximum imposed by the mission PRF. The number of runs is 50 and the SNR is $-6dB$. The achieved results are good for most of the applications and enable us to retrieve the frequency interval where the Doppler-shift belongs. More accurate methods can afterwards be used to retrieve the residual velocity inside this interval such as [2].

4. CONCLUSIONS

In this work we have shown that it is possible to estimate the Doppler-shift induced by a moving target using undersampled SAR raw-data in the azimuth spectra. We exploited the linear dependency of the Doppler-shift on the fast-time frequency. We have shown that although the static ground returns are incorrelated in the frequency domain, the same is not true for a moving target. Using this knowledge we developed a simple scheme to retrieve the true Doppler-shift induced by a moving target. Good results were shown using a combination of real and simulated SAR data for moving objects with range speeds up to 15 times above the maximum imposed by the used Pulse Repetition Frequency.
Appendix

In this appendix we want to compute the correlation of a received signal from a moving target for two different fast-time frequencies \( k \) and \( k + \Delta k \).

The received signal from a moving target in the \((k_u, k)\) frequency domains, after pulse compression is [2]

\[
S_m(k_u, k) = \left| P(\omega) \right|^2 A(k_u, k, \theta) f e^{-j\sqrt{4\pi^2 - (\frac{\omega}{c})^2} X} e^{-j(\frac{\omega}{c})^2 Y},
\]

where function \( P(\omega) \) is the Fourier transform of the transmitted signal, \( f \) is the moving target complex reflectivity, \((X, Y)\) the motion transformed coordinates [7] and \( \alpha = \sqrt{\mu^2 + \nu^2} \). The antenna radiation pattern in the \((k_u, k)\) domain is denoted \( A(k_u, k, \theta) \), where \( \theta = (\mu, \nu) \). The autocorrelation function for two different fast-time frequency values \( k \) and \( k + \Delta k \) is

\[
R_{SS_m}(k, k + \Delta k, k_u) = \int S_m(k_u, k) S_m^*(k_u - k', k + \Delta k) dk_u.
\]

(6)

Considering that \( A(k_u, k + \Delta k, \theta) = A(k_u - 2\Delta k \mu, k, \theta) \) (see (2)), and after some simple but lengthily algebraic manipulation, expression (6) can be written as

\[
R_{SS_m}(k, k + \Delta k, k_u) =
\left| P(\omega) \right|^2 f^2 e^{-j(\sqrt{\frac{4\pi^2 - (\frac{\omega}{c})^2}{\mu^2 + \nu^2}} X)}
\times \int A(k_u, k, \theta) A^*(k_u - 2\Delta k \mu - k'_u, \theta) e^{j\frac{2\pi k'_u}{c(\mu^2 + \nu^2)}} X \ dk_u.
\]

(7)

In (7) a correlation between the antenna pattern \( A(k_u, k, \theta) \) and the same function delayed by \( 2\Delta k \mu \) is easily recognized, although it is modulated by \( e^{j\frac{2\pi k'_u}{c(\mu^2 + \nu^2)}} \).

If the phase \( \frac{2\pi k'_u}{c(\mu^2 + \nu^2)} X < \pi \) one should expect that \( R_{SS_m}(k, k + \Delta k, k_u) \) exhibits a maximum which is linearly dependent of the fast-time frequency by a factor equal to \( 2\Delta k \mu \). This phase value can be made negligible by considering the typical values of \( k_u \) and \( k \) and that we can compensate the dependency of \( X \) by using the knowledge of the target area approximate range coordinates.

5. REFERENCES


