OPTIMAL M-QAM/DAPSK ALLOCATION IN NARROWBAND OFDM RADIO CHANNELS

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Abstract – This paper proposes and formulates a mathematical optimization problem in the context of multi-carrier communications and seeks for a solution. A particular multi-carrier modulation system is the orthogonal Frequency Division Multiplexing (ODFM) where modulations are implemented through a single Fast Fourier Transformer (FFT). A wireless communication system may use tens of thousands of orthogonal modulators. Given a finite set of possible digital modulators M-QAM or M-DAPSK and under certain constrains, the solution of the optimization problem should provide the optimum value of M.

Keywords – Digital Television, Integer Optimization Optimal Multi-carrier, Adaptive Joint Source-modulation.

I. INTRODUCTION

Broadcasting is moving into a digital era allowing new and enriched services and applications. The broadcasting quality is improved significantly by using multicarrier systems. The first systems using MCM (Multi-Carrier Modulation) were military HF radio links in the late 1950s and early 1960s. OFDM, a special form of MCM was patented by R.W. Chang in the US in 1970. OFDM removed the bank of steep bandpass filters that completely separated the spectrum of individual subcarriers. Orthogonality of OFDM carriers allows subcarrier spectra overlapping without inter-carrier interference (ICI). The most popular wireless broadcasting systems making use of OFDM are Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB). OFDM is nowadays efficiently implemented by applying the IDFT/IFFT at the emitter,

$$x_n = \frac{1}{N}\sum_{k=0+c}^{N-1+c} X_k e^{j\frac{2\delta}{N}kn} \qquad ,0+d \le n \le N-1+d\,, \quad (1)$$

and the DFT/FFT at the receiver,

$$X_k = \sum_{n=0+a}^{N-1+a} x_n e^{-j\frac{2\delta}{N}kn} \qquad ,0+b \le k \le N-1+b \quad (2)$$

where a, b, c and d can be any integer. For sake of simplicity, let us assume them equal to zero. X_k, k=0,1,…N-1 are integer complex numbers and represent the information to be transmitted to the receiver. As expressed in (1), X_k is modulated/multiplied by a complex exponential carrier and through the summation converted into a new discrete complex sequence x_n, n=1,2,...,N usually called a symbol. This sequence is delivered to the receiver suffering channel impairments. Thus x_n is changed by the channel, resulting in,

$$r_n = x_n \otimes h_n + w_n \qquad n = 0,1,\ldots, N-1 \quad (3)$$

where ⊗ denotes the convolution operation, h_n is a exponential decaying function and w_n is a zero mean complex Gaussian independent variable. All variables in (3) as well as X_n are random processes.

From (3), we have [1],

$$r_n = \text{IFFT}\{\text{FFT}(x_n).\text{FFT}(h_n)\} + w_n \quad (4)$$

resulting in,

$$r_n = \text{IFFT}\{X_n H_n\} + w_n \quad (5)$$

By applying the FFT to (5), we obtain,

$$R_n = X_n H_n + Z_n \quad (6)$$

with

$$Z_n = \text{FFT}(w_n)\ , \quad (7)$$

representing a zero mean complex Gaussian independent random variable. The receiver performance is measured by its capability of removing H_n and Z_n effects in (6) and thus approaching R_n to X_n. The impairments caused by H_n and Z_n are greater and greater as M_n increases. However, the greater M_n is, the more information is delivered to the destination. Therefore a tradeoff is require to find out the best values of M_n, n=0,1,…,N-1.

II. THE OPTIMIZATION PROBLEM

We will confine our problem formulation to M-QAM with M=0,4,16,32. The impairments mentioned in the above section are modeled by the bit error probability P_{M_N}. In consequence, we are interested in minimizing the following objective function defined implicitly,

$$\sum_{n=0}^{N-1} P_{M_n}(\bar{\gamma}) \quad (8)$$

where, for instance, for M=16 [2,3],

$$P_{16}(\bar{\gamma}) = \int_0^{\infty}\left[\frac{3}{4}Q\left(\sqrt{\frac{\gamma}{5}}\right)+\frac{1}{2}Q\left(\sqrt{\frac{9\gamma}{5}}\right)-\frac{1}{4}Q\left(\sqrt{5\gamma}\right)\right]\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}d\gamma$$
$$= \frac{3}{8}\left(1-\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+10}}\right)+\frac{1}{4}\left(1-\sqrt{\frac{9\bar{\gamma}}{9\bar{\gamma}+10}}\right)-\frac{1}{8}\left(1-\sqrt{\frac{5\bar{\gamma}}{5\bar{\gamma}+2}}\right) \quad (9)$$

where γ follows a chi-square distribution with two degrees of freedom and average $\bar{\gamma}$ [4]. Equation (9) takes into account the effects of H_n and Z_n described in Section I and therefore $\bar{\gamma}$ is a function of n. In (9),

$$Q(x) = \int_x^{\infty}\frac{\exp(-\frac{y^2}{2})}{\sqrt{2\pi}}dy\,. \quad (10)$$

The solution is the vector M_n, n=0,1,…, N-1, in which each element can be one from the integer set {0, 4, 16, 32}. Beyond of this constrain, the objective function (8) is subject to other constrain,

$$\sum_{n=0}^{N-1} M_n = N\overline{M} = N\frac{1}{4}(0+4+16+32) = 13N \quad (11)$$

It would be interesting to find out a closed form solution for some particular functions $\bar{\gamma}_n$ (Gaussian, Laplacian).

Other problems could have been formulated. For instance, instead of minimizing the total error probability (8) and constraining on the average bit error rate (11), we could formulate the problem by maximizing the total bit error rate and a inequality constrain, the average error probability greater than a pre-defined value.

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