COST MINIMIZATION OF A MULTIPLE SECTION ENERGY CABLE SUPPLYING REMOTE TELECOM EQUIPMENTS

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Abstract: A single cable with different sections is used to supply several remote telecom equipments. Using power supplies with a broad range of input voltage it is possible to have large voltage drops along the cable, and consequently to minimize the cable cost. As the power supplies operate at constant power and input voltage is variable along the cable, the minimization of cable material and cost is a problem of non-linear optimization.

Key words: Non-linear optimization

I. INTRODUCTION: THE PROBLEM

A long cable (two wire) is supplied in one end by a constant voltage generator (V_0). It has n cable sections and n nodes. In each node is connected a constant power load p_i ($i=1...n$). The voltage drop across each cable section V_i-V_{i+1} is the product of the resistance of the cable section r_i by the current in each cable section. And this one is the sum of the further nodes currents $i_{i+1}+ i_{i+2}+ i_{i+3}+...+ i_n$. The load current, i_i, in each node is dependent of the node voltage ($i_i = p_i / V_i$). The minimum operating voltage in the last load p_n is also known, and is aV_0 ($a<1$).

II. AN ENGINEERING SOLUTION TOWARDS THE MINIMUM COST

The author developed an algorithm to achieve a good solution for the array s_i, that seems to be the array associated to the minimum cable cost. However, the author is not able to validate the solution, neither knows if the solution is a local minimum or not.

Considering the following variables:
- total cable length:

$$L = \sum_{i=1}^{n} l_i \qquad (7)$$

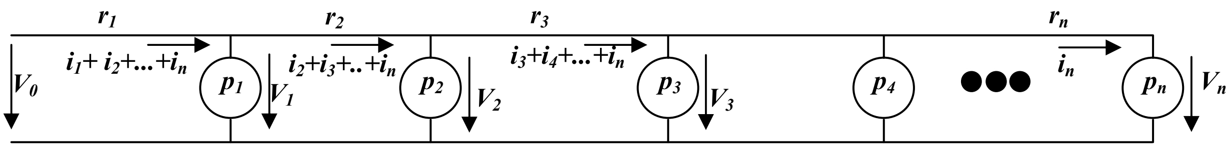

Fig. 1. Cable supplying several constant power loads

The system equations are:

$$i_i = p_i / V_i \qquad i=1 ... n \qquad (1)$$

$$V_i = V_{i-1} - r_i . \sum_{j=i}^{n} i_j \qquad (2)$$

$$V_n = a . V_0 \qquad (3)$$

The resistance of each cable section r_i depends on the known length of the section, $2 . l_i$, and copper cross section of the cable, s_i :

$$r_i = 2.K . l_i / s_i \qquad (4)$$

where K is a known constant. The cost of each cable section depends on the volume of copper material $2 . l_i . s_i$:

$$c_i = 2. c . l_i . s_i \qquad (5)$$

where c is a constant. The total cable cost is:

$$C = \sum_{i=1}^{n} c_i \qquad (6)$$

We know all the values p_i and l_i and the constants a ,V_0 , K and c. We need to calculate the values of the cross sections s_i that minimize the total cable cost C.

- total power of distributed loads:

$$P = \sum_{i=1}^{n} p_i \qquad (8)$$

Considering a single cable with a length L and a single load with the total power P, at the end of the cable, operating at the voltage $a.V_0$, we calculate the required cable cross section as follows:

$$i = \frac{P}{a.V_0} \qquad (9)$$

$$r = \frac{V_0 - a.V_0}{i} = \frac{a.(1-a).V_0^2}{P} \qquad (10)$$

$$s = \frac{2.K.l}{r} = \frac{2.K.L.P}{a.(1-a).V_0^2} \qquad (11)$$

We may introduce another variable, called "reference cross section", S:

$$S = \frac{s}{i} = \frac{2.K.L}{(1-a).V_0} \qquad (12)$$

The solution for the minimum cost is:

$$s_i = S . \sum_{j=i}^{n} i_j \qquad (13)$$

III. NUMERICAL EXAMPLE

Considering the following data:
n=5
l=[2, 3, 4, 1, 2]

p=[1000, 500, 1500, 1400, 600]
V_0= 500
a= 0.6
and an arbitrary value for ***2.K=1***, the value of the reference cross section is obtained from (12), ***S= 0.06***, with ***L=12*** and ***P=5000***. Using equations (1) to (4) and (13), it is possible to calculate the values of the voltages in the nodes, v_i , the currents in the cable sections, i_i , and the cable cross sections, s_i :

$V_5 = a \,.\, V_0 = 300$
$i_5 = p_5 / V_5 = 2$
$s_5 = S \,.\, i_5 = 0.12$
$r_5 = 2 \,.\, K \,.\, l_5 \;/\; s_5 = 16.67$

$V_4 = V_5 + r_5 \,.\, i_5 = 333.33$
$i_4 = p_4 / V_4 = 4.2$
$s_4 = S \,.\, (i_4 + i_5) = 0.372$
$r_4 = 2 \,.\, K \,.\, l_4 \;/\; s_4 = 2.688$

$V_3 = V_4 + r_4 \,.\, (i_4 + i_5) = 350$
$i_3 = p_3 / V_3 = 4.286$
$s_3 = S \,.\, (i_3 + i_4 + i_5) = 0.629$
$r_3 = 2 \,.\, K \,.\, l_3 \;/\; s_3 = 6.358$

$V_2 = V_3 + r_3 \,.\, (i_3 + i_4 + i_5) = 416.7$
$i_2 = p_2 / V_2 = 1.2$
$s_2 = S \,.\, (i_2 + i_3 + i_4 + i_5) = 0.7012$
$r_2 = 2 \,.\, K \,.\, l_2 \;/\; s_2 = 4.279$

$V_1 = V_2 + r_2 \,.\, (i_2 + i_3 + i_4 + i_5) = 466.7$
$i_1 = p_1 / V_1 = 2.143$
$s_1 = S \,.\, (i_1 + i_2 + i_3 + i_4 + i_5) = 0.830$
$r_1 = 2 \,.\, K \,.\, l_1 \;/\; s_1 = 2.41$

and verifying the value of V_0, is:
$V_0 = V_1 + r_1 \,.\, (i_1 + i_2 + i_3 + i_4 + i_5) = 500$
The calculation of cable cost is:
$C = 2.\, c \,.(l_1 \,.\, s_1 + l_2 \,.\, s_2 + l_3 \,.\, s_3 + l_4 \,.\, s_4 + l_5 \,.\, s_5) = 2.\, c \times 6.8916$
The author tried several small variations of the calculated cross sections and obtained always an increase of the cost function C.