OPTIMAL GLOBALITY IN TIME-DOMAIN DIGITAL ARMA FILTER DESIGN

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Abstract – “Classic”, local, gradient-type optimization algorithms are frequently used for finding local optima for the optimal filter design problem. By plotting the target function, it was verified that only one stationary point seems to exist in a classic time-domain optimal filter design problem. A mathematical proof of this property is required in order to establish that global optimization algorithms are unnecessary. On the other hand, if the property is violated, some sort of understanding of the function behavior is very welcome as a basis for designing an efficient global optimization procedure.

Keywords – digital signal processing, ARMA filter, optimal filter design.

I. INTRODUCTION

Digital ARMA (autoregressive moving average), linear time-invariant filters are frequently used in telecommunications. These filters accept an input sequence $x[n]$ and produce an output $y[n]$ according to the difference equation

$$\begin{aligned} y[n]+a_1y[n-1]+...+a_{na}y[n-na]= \\ =b_0x[n]+b_1x[n-1]+...+b_{nb}x[n-nb] \end{aligned} \quad (1).$$

In (1), (na,nb) is the filter order, $a_1,a_2,...,a_{na}$ are the auto-regressive coefficients, and $b_0,b_1,...,b_{nb}$ are the mooving average coefficients.

A classic filter design problem consists of establishing (na,nb) and the filter coefficients in such a way that, for a known input sequence $x[n]$, the output $y[n]$ approaches an “ideal output” $y_d[n]$. This problem is frequently formulated as

$$\min_{a\in R^{na},b\in R^{nb+1}} J(a,b,x)=\sum_{n=0}^{N-1}\frac{1}{2}\left(y[n]-y_d[n]\right)^2 \quad (2),$$

where $a=\begin{pmatrix}a_1 & a_2 & \cdots & a_{na}\end{pmatrix}$, $b=\begin{pmatrix}b_0 & b_1 & \cdots & b_{nb}\end{pmatrix}$ and N is a large integer number (usually $N\in[50,300]$). It is usual to begin the (iterative) design procedure by choosing a low filter order and then increase it until the optimal value of the target function becomes small enough.

The optimization problem (2) is non-convex and differentiable. The target function gradient and *local*, gradient-based optimization algorithms may be found in [1].

II. THE PROBLEM

We have plotted the line search function

$$\phi(\alpha)=J(a^0+\alpha s,b^0+\alpha s,x) \qquad (\alpha\in R) \qquad (3)$$

for many starting points (a^0,b^0), search directions s and filter design problems, and verified that it only has one stationary point when N is large, as required ($N\in[50,300]$).

We would like to obtain a rigorous mathematical proof of this property, or a set of examples where the property is violated, and some sort of understanding of the function behaviour .

A proof of the aforementioned property would be very welcome because it would enable us to prove that *local* optimization methods always find the *global* optimal solution of (2), and therefore that global optimization procedures are not required. On the other hand, information on the function behavior would be helpful for establishing an efficient global optimization procedure if the property turns out to be violated.

III. REFERENCES

[1] James A. Cadzow, “Recursive Digital Filter Synthesis Via Gradient Based Algorithms”, *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. ASSP-24, no. 5, October 1976, pp. 349-355.