COMPUTATIONAL COMPLEXITY OF DISCRETE FOURIER TRANSFORM

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Abstract - A mathematical theory on the computational complexity of the Discrete Fourier Transform (DFT) is required, independently of any kind of known Fast Fourier Transform (FFT) algorithm.

Keywords - DFT, FFT, computational complexity, lower bound or limit.

I. INTRODUCTION

The discrete Fourier transform (DFT) is an important mathematical tool in modern digital signal processing and telecommunications fields. The direct transform is given by

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N} \ , \qquad (1)$$

for, $0 \le k \le N-1$, and where N is the length of the sequence $x[n]$ (real or complex). Usually, N is a power of 2, 4, 8 or 16.

In the last decades, several families of fast computational DFT algorithms (FFT) have been developed mainly based on the properties of the complex operator in (1). Some examples are the radix n [1,2,3], the split-radix [1] and its extensions [4], the prime factors decompositions [1,2,3] and the hardware/software optimised decompositions algorithms [5] .

II. THE PROBLEM

Independently of any kind of known efficient algorithm, the proposed challenge is to develop a mathematical theory on the computational complexity of the DFT, which leads to lower bounds or limits (in case of existence) on the number of elementary operations (real or complex non-trivial additions and multiplications) necessary to compute (1) as functions of the sequence length N.

We hope to find a boundary in terms of elementary operations that is impossible to go beyond. Some introductory and related material can be found in [1,2,3,6].

Simultaneously, this knowledge will allow us to verify the level of optimality of the available algorithms and if there is room for further research on improvement and development of entirely new fast algorithms.

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