NUMERIC INTEGRATION OF RAPIDLY OSCILLATING FUNCTIONS

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Abstract – Numeric integration of rapidly oscillating functions presents considerable difficulties. A method to deal with this problem when the integrand is the product of a function by an oscillating function of the type sine, cosine or Bessel function is sought.

Keywords – Numeric integration, oscillating function, sine, cosine, Bessel function.

I. INTRODUCTION

Surface integrals of rapidly oscillating functions are a common problem in the calculation of radiation pattern of aperture antennas. In some cases it is possible to perform one of the integrals analytically, a first (important) step in solving the problem. In many cases of practical interest the remaining integral may be evaluated using an adaptive Gauss quadrature without undue difficulty. However, when the aperture size is large compared with the wavelength, the use of a Gauss quadrature may not be possible either because of the time involved or because of error accumulation.

The problem may be formulated as the calculation of:

$$\int_0^{2p} f(x) g(ax)\, dx ,$$

where g is an oscillating function: either a sine, a cosine or a Bessel function of the first kind, a is a large real number and $f(x)$ is a well behaved, continuous function, with continuous derivatives.

II. SPECIAL CASES

For a few special cases, for instance when $f(x)$ is a polynomial the integral may be solved analytically. However in the case of Bessel functions of the first kind the result is expressed in terms of hypergeometric functions which may take a long time to evaluate or the result may be seriously in error.

In some cases $f(x)$ may stepwise be approximated by a polynomial, that is:

$$f(x) = p_i(x)$$

for

$$b_i \le x \le b_{i+1},$$

where $p_i(x)$ is a polynomial in x. In such cases it is possible to express the integral as a sum of integral each of which may be calculated analytically as

$$\int_0^{2p} f(x)\, g(ax)\, dx = \sum_{i=1}^{n} \int_{b_i}^{b_{i+1}} p_i(x) g(ax)\, dx .$$

III. THE GENERAL CASE

In the general case, that is when $f(x)$ cannot easily fit in one of the previous cases, for instance when $f(x)$ is itself an oscillatory function then it the “brute force” solution, implies, for instance, using Simpson’s rule with 4 to 5 integration points per period of the fastest oscillatory function. This approach produces acceptable results but, in practice, is limited to about $a \le 10^6$ or less, in case $f(x)$ is also oscillatory.

Obviously the “brute force” approach is not ideal and a better solution is required.