Mode Transformers for Soft and Hard Surface Waveguides by using Chiral Material

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Abstract

The field propagation in soft and hard surface waveguide, also called balanced corrugated waveguide, filled with chiral medium is considered. The eigenfields inside the waveguide are circularly polarized and are propagating separately. For small chirality parameter values the eigenfields are slightly coupled and the polarization of the propagating field is changed. This effect causes a mode transformation between TM and TE modes.

1. Introduction

Wave propagation in cylindrical corrugated waveguide filled with chiral medium is considered. The corrugation on the surface of the waveguide is such that the boundary condition is equal to the soft and hard surface boundary [1]. These kind of waveguides are used, for example, in antenna feed horns. It is known that the eigenfields associated to the soft and hard surfaces are circularly polarized. Also the eigenwaves propagating in chiral medium are circularly polarized, denoted by $+$ and $-$ waves [2] - [4]. When the corrugation in a waveguide structure is in transverse direction the waveguide is called soft surface waveguide, and when the corrugation is in axial direction it is called hard surface waveguide [1]. Inside the chiral soft and hard surface waveguide there are propagating $+$ and $-$ waves separately. However, when the chirality parameter of the medium is very small these two eigenwaves are propagating almost with the same propagation factor which results in a change in polarization of the propagating field.

2. Theory

The electric and magnetic fields in waveguide depend on z as $e^{-j\beta z}$ where β is the propagation factor. The waveguide is filled with chiral material with the constitutive relations [4]

$$\mathbf{D} = \varepsilon\mathbf{E} - j\kappa\sqrt{\mu_o\varepsilon_o}\,\mathbf{H}, \qquad \mathbf{B} = \mu\mathbf{H} + j\kappa\sqrt{\mu_o\varepsilon_o}\,\mathbf{E}, \tag{1}$$

where ε, μ and κ are permittivity, permeability and the chirality parameter of the medium, respectively. In the waveguide structure the electric and magnetic fields are written with transverse and axial parts as

$$\mathbf{E} = \mathbf{e} + E_z\mathbf{u}_z, \qquad \mathbf{H} = \mathbf{h} + H_z\mathbf{u}_z, \tag{2}$$

which are inserted into the Maxwell equations

$$\nabla\times\mathbf{E} = -j\omega\mathbf{B}, \qquad \nabla\times\mathbf{H} = j\omega\mathbf{D}. \tag{3}$$

On the other hand, the fields in chiral medium are expressed in terms of right hand and left hand circularly polarized waves denoted by $+$ and $-$ waves. After eliminating the transverse fields $\mathbf{e}$ and $\mathbf{h}$ and using the decomposition into + and − parts the Maxwell equations reduce to the Helmholtz equation for axial field components

$$[\nabla_t^2 + k_{c\pm}^2]E_{z\pm}(\rho, \varphi) = 0, \tag{4}$$

where $E_{z\pm} = \frac{1}{2}(E_z \mp j\eta H_z)$, $k_{c\pm} = \sqrt{k_\pm^2 - \beta^2}$ and $k_\pm = k \pm k_o\kappa$. The ∇_t operator is the transverse part of ∇. The general solutions of the Helmholtz equation in cylindrical coordinates are Bessel functions of the first kind

$$E_{z\pm}(\rho, \varphi) = A_{n\pm}J_n(k_{c\pm}\rho)e^{jn\varphi}. \tag{5}$$

The all other field components can be expressed with these two axial field components. The partial transverse fields are obtained from the axial parts as

$$\mathbf{e}_\pm = [-\frac{j\beta}{k_{c\pm}^2}\nabla_t \mp \frac{k_\pm}{k_{c\pm}^2}\mathbf{u}_z \times \nabla_t]E_{z\pm}. \tag{6}$$

The coefficients $k_{c\pm}$ and $A_{n\pm}$ are determined by the boundary and initial conditions.

The two partial transverse fields $\mathbf{e}_+$ and $\mathbf{e}_-$ are elliptically polarized of opposite handedness with respect to the direction of propagation. The total fields are obtained as a combination of transverse and axial + and − waves as

$$\mathbf{E} = \mathbf{e}_+ + \mathbf{e}_- + (E_{z+} + E_{z-})\mathbf{u}_z \tag{7}$$

and

$$\mathbf{H} = \frac{j}{\eta}[\mathbf{e}_+ - \mathbf{e}_- + (E_{z+} - E_{z-})\mathbf{u}_z]. \tag{8}$$

The parameters $k_{c\pm}$ are determined by the boundary condition for the soft and hard surface at $\rho = a$

$$\mathbf{u} \cdot \mathbf{E} = 0, \qquad \mathbf{u} \cdot \mathbf{H} = 0, \tag{9}$$

where $\mathbf{u} = \mathbf{u}_\varphi$ for soft surface and $\mathbf{u} = \mathbf{u}_z$ for hard surface boundary [1]. For the soft surface waveguide with index $n = 0$ and for the hard surface waveguide with all index n the boundary conditions (9) lead to the eigenvalue equation [5]

$$J_n(k_{c+}a)\, J_n(k_{c-}a) = 0. \tag{10}$$

The solution of the eigenvalue equation is

$$k_{c\pm} = \frac{p_{ns}}{a}, \tag{11}$$

where p_{ns} are the zeros of the Bessel functions. The propagation factors for + and − waves are now obtained

$$\beta_\pm = \sqrt{k_\pm^2 - \left(\frac{p_{ns}}{a}\right)^2}. \tag{12}$$

The value of the propagation factor is different for + and − waves when the waveguide is filled with chiral material. In a nonchiral case these two values reduce to $\beta = \sqrt{k^2 - (p_{ns}/a)^2}$.

3. Mode Transformation

The polarization properties of the propagating fields inside the chiral soft and hard surface waveguides are considered when the value of the chirality parameter of the medium is small. It will be shown that the small chirality value affects mainly to the polarization of the propagating field. Chiral media can be fabricated by incerting small helices into the base material and the chirality parameter is proportional to the density of chiral inclusions. High density of inclusions increases losses [6]. Now, the requered chirality parameter is small and is achieved with small density of inclusions and also the losses are small.

Denoting the wave numbers of partial waves in chiral medium as

$$k_\pm = k(1 \pm \kappa_r), \tag{13}$$

where $\kappa_r = \kappa\sqrt{\frac{\mu_o\varepsilon_o}{\mu\varepsilon}}$ and assuming that $|\kappa_r| \ll 1$, the propagation factors are approximated as

$$\beta_\pm = \sqrt{k^2(1 \pm \kappa_r)^2 - \left(\frac{p_{ns}}{a}\right)^2} \approx \beta \pm \frac{k^2}{\beta}\kappa_r. \tag{14}$$

Here, the parameters $\beta = \sqrt{k^2 - k_c^2}$ and $k_c = \frac{p_{ns}}{a}$ are the same as for a nonchiral waveguide. The axial field components reduce now to the form

$$E_{z\pm}(\rho, \varphi, z) \approx A_{n\pm} J_n(k_c\rho) e^{jn\varphi} e^{\mp j\frac{k^2}{\beta}\kappa_r z} e^{-j\beta z} \tag{15}$$

and, similarly, the transverse partial waves are approximated as

$$\mathbf{e}_\pm(\rho, \varphi, z) \approx [-\frac{j\beta}{k_c^2}\nabla_t \mp \frac{k}{k_c^2}\mathbf{u}_z \times \nabla_t] E_{z\pm}(\rho, \varphi, z). \tag{16}$$

The total axial field components are

$$\begin{aligned} E_z &= E_{z+} + E_{z-} \\ &= [(A_{n+} + A_{n-})\cos(\frac{k^2}{\beta}\kappa_r z) - j(A_{n+} - A_{n-})\sin(\frac{k^2}{\beta}\kappa_r z)] J_n(k_c\rho) e^{jn\varphi} e^{-j\beta z} \end{aligned} \tag{17}$$

and

$$\begin{aligned} H_z &= \frac{j}{\eta}[E_{z+} - E_{z-}] \\ &= \frac{j}{\eta}[(A_{n+} - A_{n-})\cos(\frac{k^2}{\beta}\kappa_r z) - j(A_{n+} + A_{n-})\sin(\frac{k^2}{\beta}\kappa_r z)] J_n(k_c\rho) e^{jn\varphi} e^{-j\beta z}. \end{aligned} \tag{18}$$

In these expressions the fields are written in terms of right hand and left hand circularly polarized components. The general field inside a waveguide can also be presented as a combination of TE and TM fields. Denoting at $z = 0$ the axial field components as

$$E_z(0) = A_{n+} + A_{n-} = E_n \tag{19}$$

and

$$H_z(0) = \frac{j}{\eta}[A_{n+} - A_{n-}] = H_n, \tag{20}$$

the axial electric field is

$$E_z(z) = [E_n \cos(\frac{k^2}{\beta}\kappa_r z) - \eta H_n \sin(\frac{k^2}{\beta}\kappa_r z)] J_n(k_c\rho) e^{jn\varphi} e^{-j\beta z} \tag{21}$$

and the axial magnetic field is

$$H_z(z) = [H_n \cos(\frac{k^2}{\beta}\kappa_r z) + \frac{E_n}{\eta}\sin(\frac{k^2}{\beta}\kappa_r z)]J_n(k_c\rho)e^{jn\varphi}e^{-j\beta z}. \tag{22}$$

The coefficients E_n and H_n are determined by the initial conditions.

The mode transformation effect is clearly seen by considering the axial field components as a function of z. If, for example, at $z = 0$ we have $H_n = 0$, there exists the axial electric field but no axial magnetic field, we have then TM_{ns} fields. At the distance

$$z = \frac{\pi\beta}{2k^2\kappa_r}, \tag{23}$$

which is denoted by $\lambda_p/4$, there exists the axial magnetic field but no axial electric field which means that we have TE_{ns} fields. So, after the distance $z = \lambda_p/4$ the original TM mode is changed to TE mode.

When the length of the chiral soft and hard surface waveguide is twice the value of (23), denoted by $\lambda_p/2$, the field configuration is at 180° phase shift of its original value. This means that the chiral soft and hard surface waveguide of length $\lambda_p/2$ works as a 180° phase shifter. In a general case, inside the soft and hard surface waveguide there can propagate a hybrid mode. With other choice of the length for the waveguide as done in the previous examples the original hybrid mode propagating inside the waveguide can be transformed to another hybrid mode.

4. Conclusion

The mode transformation effect in chiral soft and hard surface cylindrical waveguide are considered. The soft and hard surface waveguide is filled with slightly chiral material. The small chirality affects to the polarization of the propagating field. The eigenvalue equation for the chiral hard surface waveguide is evaluated. Also, the eigenvalue equation for the chiral soft surface waveguide for the spherically symmetric mode is similar as for the hard surface waveguide. It is demonstrated that with a proper length of the chiral soft and hard surface waveguide the TM mode is changed to TE mode and vice versa. Also when the length is twice of this proper length the original field pattern is suffered a 180° phase shift. This kind of mode transformers and phase shifters may be used as matching elements between different kind of waveguides or between waveguides and antennas.

References

[1] P-S. Kildal, " Artificially soft and hard surfaces in electromagnetics," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 10, pp. 1537–1544, 1990.

[2] P. Pelet and N. Engheta, "The theory of chirowaveguides," *IEEE Trans. Antennas Propagation,* vol. 38, no. 1, pp. 90–97, January, 1990.

[3] S. F. Mahmoud, "Mode characteristics in chirowaveguides with constant impedance walls," *J. Electromag. Waves Appl.,* vol. 6, no. 5/6, pp. 625–640, 1992.

[4] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media.* Norwood, NY: Artech House, 1994.

[5] A. J. Viitanen, "Chiral hard-surface waveguide mode transformer," *IEEE Trans. Microwave Theory Tech.*, vol. 48, no. 6, June 2000, to appear.

[6] F. Mariotte, S. A. Tretyakov, and B. Sauviac, "Isotropic chiral composite modeling: Comparison between analytical, numerical, and experimental results," *Microwave Opt. Technol. Lett.,* vol. 7, no. 18, pp. 861–864, 1994.