Modelling Photonic Bandgap Structures using FDTD

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Abstract

FDTD modelling of dielectric PBG structures is considered. In this paper PBGs are assumed to be related to layered structures which are especially used in optical waveguiding applications. Many essential features can be modelled 2-dimensionally.

1. Introduction

Photonic bandgap (PBG) material is periodically inhomogeneous material, in which electromagnetic waves having frequency in the bandgap range, can't propagate. Due to the periodical structure, PBGs are often called photonic crystals, too [1]. An example of 2-D PBG material is a uniform lattice of cylindrical air holes inside dielectric host material. In real 3-D world this might correspond to a silicon plate having cylindrical air holes. Properly manufactured PBG material can be used as a frequency selective reflective surface. Potential applications are highly efficient optical lasers and sharp bends in optical waveguides, for example. In this paper purely dielectric PBG structures are considered, even though a PBG can also be manufactured using metal rods. The chosen PBG lattice is triangular, formed by cylindrical air holes in dielectric host medium (figure 1). The main interest is in 2-D modelling of the PBG waveguides and waveguide bends.

The behaviour of EM fields in space-time in a PBG structure can be modelled using FDTD (Finite-Difference Time-Domain) which essentially means solving Maxwell's equations in discretized space and time co-ordinates [2]. FDTD has been widely used in PBG research during the last few years, see [3], [4], for example.

2. Modelling PBG Waveguides

Modelling PBG structures requires some important properties from the FDTD software used:

- model for a dielectric cylinder
- freedom to describe complicated excitation fields i.e. the time-domain behaviour and the field distribution in space.
- possibility to store "measured" fields, at certain points and time intervals, to hard disk. Stored time-dependent fields are analyzed later (F-transforms, power flow analysis etc.)

The modelling scheme in general can be considered as a "virtual measurement setup". There must be a source i.e. a field excitation, for which time and spatial dependence is properly chosen. The space-time behaviour of EM fields is governed by Maxwell's equations which are solved using FDTD. Also, there must be measurement surfaces, on which time-dependent fields are sampled and stored to hard disk for later analysis. Figure 1 shows measurement setup involving straight PBG waveguide.

When analyzing the performance of a guiding structure, computing power flow, instead of just measuring field values, should be preferred. Just inspecting field strenght does not tell, how well the power is actually propagating. For example, with a standing or evanescent wave, field value near the output can be $\neq 0$ but power is not propagating.

Power flow is computed by spatially integrating the normal poynting vector $\mathbf{S}$. The integration is done over a surface (3-D simulation) or over a line (2-D). Using simple and fast trapezoidal rule integration leads to weighted sum of poynting vector values. For example, in 2-D, when computing power flow in $+x$-direction through measurement line $M1$ ($x = x_1$, $y \in [y_1, y_2]$, see figure 1), one uses stored fields $E_y(x_1, y, t)$ and $H_z(x_1, y, t)$. Poynting $S_x(x_1, y, t) = E_y(x_1, y, t)H_z(x_1, y, t)$ and time-dependent power flow (per-unit-length) is

$$P(t) = \int_{y_1}^{y_2} S_x(x_1, y, t)dy \approx \Delta y \left[\sum_{j=2}^{N-1} S_{x,j}(t) + \frac{1}{2}\left(S_{x,1}(t) + S_{x,N}(t)\right) \right], \quad y_2 - y_1 = (N-1)\Delta y, \quad (1)$$

where the approximation corresponds to trapezoidal rule integration, and Δy is FDTD cell-size.

By computing power flow in frequency domain, $P(f)$, one can effectively obtain, for example, power transmission $T(f) = P_2(f)/P_1(f)$ for a certain waveguide bend. This requires stored fields $E_i(r_i, t)$ and $H_i(r_i, t)$, $i = 1...M$, where M is the number of measurement planes (often $M = 2$). Performing Fourier transform from time to frequency domain one gets $E_i(r_i, f)$ and $H_i(r_i, f)$. Then for every point r_i (essential field components of E and H depend on the plane i orientation),

$$S(r_i, f) = \frac{1}{2}\Re[E(r_i, f)\ H^*(r_i, f)] \quad (2)$$

is computed. Via integration of S, one gets power flow through measurement planes, $P_i(f)$, $i = 1...M$. Power flow analysis in frequency domain is efficient, because only one simulation, using pulse-like excitation, is needed to get the transmission $T(f)$.

The absolute values are not needed for power flow at different frequencies or locations, because only the spectrum of power flow matters or $T(f) = P2(f)/P1(f)$, for example. Hence, formulas such as 1 and 2, are used without coefficients Δy and $1/2$. Such simplifications are used also with 3-D analysis. Also, the P(f)-functions are divided by the power spectrum of the input signal, to actually see the filtering effect caused by the waveguide itself.

In the formulas above it was quietly assumed that E, H and S sharing the same co-ordinate (x, y, t) are also physically at the same point in space-time, which is not exactly true due to Yee's FDTD cell [2]. Additionally to numerical dispersion error, also this may cause some error at higher frequency range.

3. Numerical Examples

The numerical examples are done in 2-D and assuming TE_z polarisation i.e. $\mathbf{H} = H(x, y, t)\mathbf{u}_z$ and $\mathbf{E} = E_x(x, y, t)\mathbf{u}_x + E_y(x, y, t)\mathbf{u}_y$.

First example is a power flow analysis of a straight PBG waveguide (linear lattice defect). Figure 2 shows a H_z-field snapshot at one time instant and figure 3 (left) $P(f)$ at one measurement plane, with waveguide width w as a varying parameter. PBG material remains same i.e. frequency bandgap is same in every case. When f belongs to bandgap, PBG works as a reflecting surface or as a boundary as far as energy flow is conserned. A mode can't propagate power in the waveguide, if $f < f_{cut-off}$. Cut-off frequency goes up as the waveguide is made narrower. This effect can be seen in figure 3. It may be practical solution to determine the channel width so that only fundamental mode propagates in the bandgap frequency region.

Second example deals with determining the wavelength $\lambda(f)$ of a propagating wave, in a straight PBG waveguide which can be approximately considered as a lossless periodic waveguide, when f is in the bandgap (small radiation loss). As in example 1, narrow-band pulse-like excitation was used, and F-transformed fields at two planes were computed. Using the field phase-difference between the planes (plane separation was a i.e. the waveguide period), wavelength $\lambda(f)$ was obtained. Figure 3 (right) shows the plot $\lambda(f)$, again w as a parameter. The results were checked at some frequencies using time-harmonic excitation.

Note that in examples 1 and 2 a PEC-boundary at $y = 0$ (figure 2) was used to exclude the antisymmetric modes. So, when f is in the bandgap and small enough, the field roughly corresponds to fundamental mode. Willing to investigate antisymmetric modes, one should use PMC-boundary.

Third example is a transmission analysis of a 60^o waveguide bend. $P(f)$ was computed at two planes. The H_z-field snapshot and transmission $T(f)$ are shown in figure 4.

Due to some unwanted reflections the field sampling time window must often be limited. This causes some decrease in accuracy in F-transformed fields, $P(f)$ and $T(f)$. Considering practical modelling and design, a complicated structure is much more easily descibed using integer-like dimensions. The results, a frequency response etc., can be easily transformed later into the frequency range of interest which corresponds to the real-world physical dimensions.

4. Conclusion

2-D modelling of PBG structures was demonstrated by way of few examples. Using pulse-like excitation and after the simulation F-transforming the fields at measurement planes, one can effectively study the features of a PBG structure. It seems that 2-D simulation is often suitable for design purposes, when dealing with layered structures where the field is mostly consentrated in one layer (PBG plate). Taking the third dimension into account often means just higher radiation loss and a bit frequency-shifted response, due to a different effective refraction index seen by the wave.

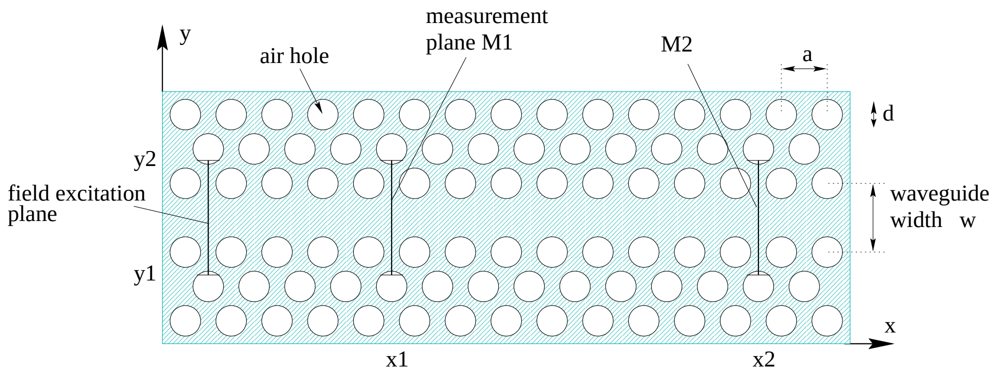

Figure 1: Triangular lattice PBG. Straight waveguide measurement setup.

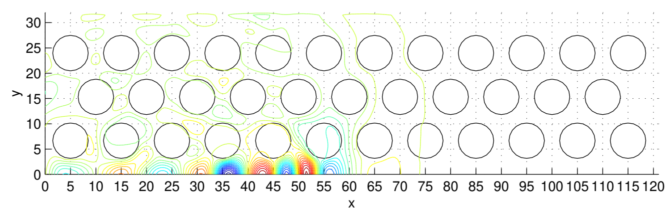

Figure 2: H_z-field snapshot. Computation domain was halved using PEC-boundary at $y = 0$. Consequently only symmetric modes can exist. The excitation field was chosen to support 1. symm. mode i.e. the fundamental mode.

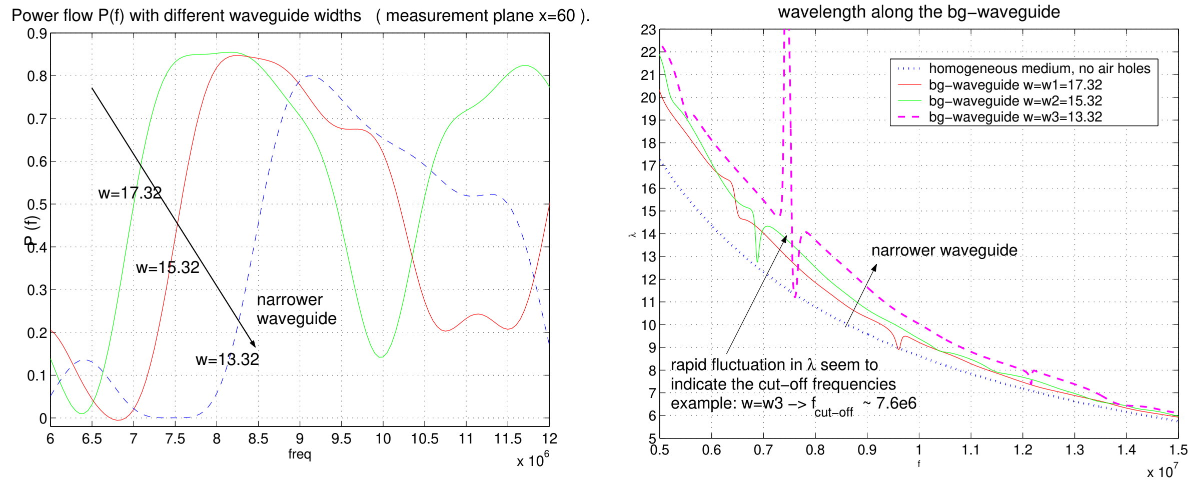

Figure 3: Left: power flow at $x = 60$. Right: wavelength in the PBG waveguide.

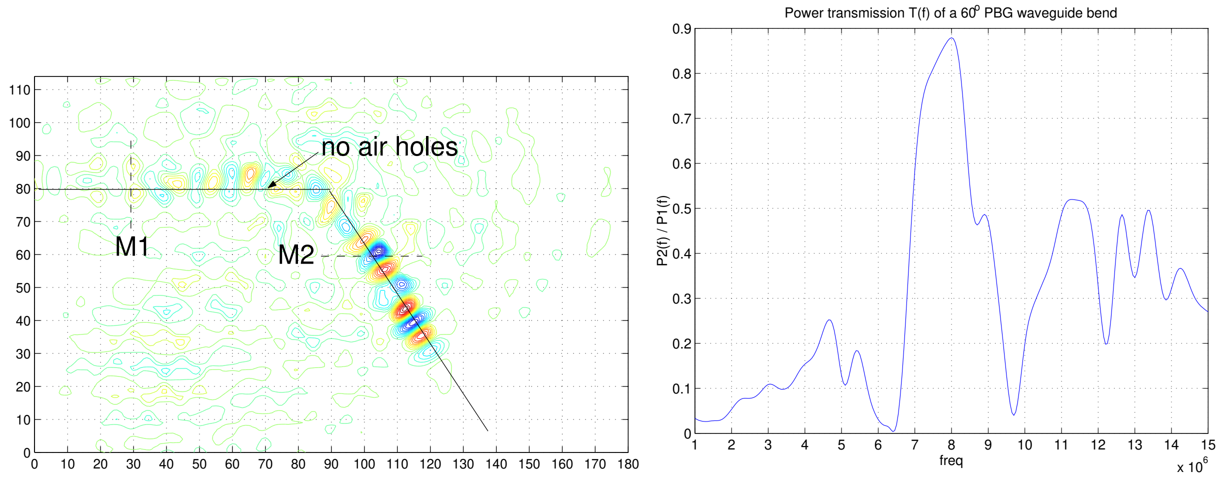

Figure 4: Left: H_z-field snapshot. Right: power transmission of a 60^o bend.

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