Fundamental Limitation on the Performance of Chiral Radar Absorbing Materials

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Abstract

We show that a fundamental limitation exists on the integral of the dB-reflection coefficient over wavelength for a passive metal-backed absorber, whose value is determined by the low-frequency behaviour only. This limit is the same for dielectric absorbers, and for chiral and omega absorbers.

1. Introduction

Standard designs of radar absorbing material on a conducting backplane become inconveniently thick for the absorption of longer wavelength radiation. Attempts to reduce the thickness of the absorber, for example by increasing the dielectric constant of the materials, often also result in a decrease of the bandwidth of good absorption, proportional to the thickness reduction achieved (figure 1).

A possible method of overcoming this problem is the use of artificial materials. Both chiral and omega materials have been proposed to provide improved absorbers. We have previously performed broadband numerical optimisations of various composite material absorbers, including helix loaded chiral composites. As a comparison, we also performed similar optimisations using a material loaded with straight dipoles or circular loops, which give a frequency-dependent dielectric constant (and permeability, for loops), but are not chiral. These optimisations incorporated a method-of-moments analysis of the polarisabilities of a single included object, to ensure that the constitutive parameters used were physically realisable. The general validity of the modelling has been tested against measurements of the constitutive parameters of various helix-loaded composites [1].

In these optimisations it was found that for a fixed layer thickness the curve of absorption with frequency was quite similar for various aspect ratios of helix, including the degenerate cases of a straight dipole and an (almost) flat loop (see figure 2). Variations of the composite allow the shape of the absorption-frequency curve to be adjusted, but do not allow the area enclosed by the curve to be increased without limit. No great advantage was found for a chiral medium [2]. Similar computations have been performed by Wallace [3] for magnetic materials: he also raised the possibility that there is fundamental limitation arising from the Kramers-Kronig relations.

To see how such a limitation might arise, consider a single-layer absorber with frequency-dependent dielectric constant $\epsilon(f)$. An ideal frequency variation can be found by computing at each frequency what value of $\epsilon(f)$ gives zero reflection for a fixed layer thickness. The real part is roughly proportional to $1/f^2$, and the imaginary part to $1/f$. This preserves phase relationships between the front and rear of the layer and attenuation through the layer. But since any real material is causal (it cannot react to an electromagnetic pulse before the pulse arrives), the real

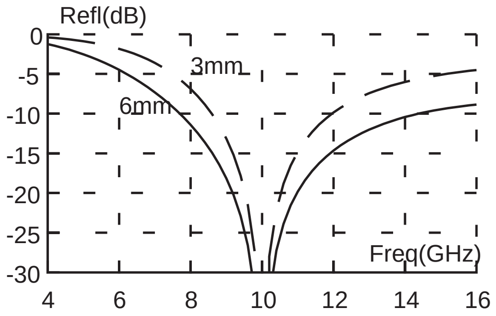

Fig. 1: Single-layer absorbers of two thicknesses

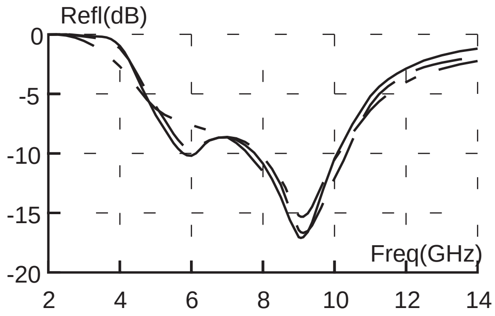

Fig. 2: Layer of two-turn helices (Full=dipole, broken=tall helix, dot-dash=flat helix)

and imaginary parts of its dielectric constant are connected by the Kramers-Kronig relations. We cannot independently specify the real and imaginary parts, as our ideal frequency variation requires, and so this variation may be unrealisable.

2. Derivation of the Limit

It has been known for a long time that a similar limitation on broadband performance exists in the design of matching circuits (e.g. [4]). We originally adapted that theory to the problem of a Salisbury screen radar absorber [5]. We construct a transmission line model of the electromagnetic problem, with a frequency-dependent shunt impedance $Z(f)$ to represent the absorbing layer, and terminated with a short circuit for the metal backplane. We also approximated the transmission line representing the spacer material by a single-stage of an LC ladder network. Using a contour integral method similar to that to be described shortly, we obtained an upper limit on the reflection coefficient of:

$$\frac{1}{f_0}\int_0^\infty |R_{dB}(f)|\,df \leq \frac{320}{\ln 10}\frac{h}{\lambda_0}\,, \tag{1}$$

where f_0 and λ_0 are the frequency and wavelength at the centre of the absorption band, and h is the thickness of the layer. A Salisbury screen has an infinite series of absorption bands (figure 3), and the integral of the dB-reflection coefficient is clearly infinite. So this limitation cannot be rigorously true. But the approximation of using only a single step of ladder network has the effect of confining attention to the first absorption band of a Salisbury screen, and the limitation does work with this restriction. We found that in all our optimisations of dielectric and chiral materials, this limit was never violated, and integrals of up to 70% of the limit could be achieved. A scaling argument shows that if the magnetic materials were involved, the limit increases by the permeability μ.

At the PIERS-98 Conference, K. N. Rozanov published an independent paper on this topic [6]. He wrote the integral over frequency rather than wavelength. This overcomes the problem of the infinite frequency integral. Thus the method will be described here in wavelength terms.

Consider a planar absorber, with a frequency-dependent field reflection coefficient $R_f(f)$, which can also be expressed as a function of wavelength $R_\lambda(\lambda)$. We calculate the integral $\int_0^\infty \ln[1/R_\lambda(\lambda)]d\lambda$ around a contour in the complex plane, consisting of the entire real axis, closed by a large-radius semi-circle in the positive imaginary half-plane. Since $R_f(f)$ is the Fourier transform of a real and causal impulse response, it is analytic in the lower half-plane, and has the symmetry $R_f(-f^*) = R_f^*(f)$. The wavelength reflection coefficient $R_\lambda(\lambda)$ has the same symmetry, and is analytic in the upper half-plane. Using the symmetry, the integral along the real axis is equal to $2\int_0^\infty \ln[1/|R_\lambda(\lambda)|]d\lambda$. This is proportional to the integral of the magnitude of the reflection coefficient measured in dB.

To compute the integral around the large semicircle, we expand $\ln[1/R_\lambda]$ as a power series $\sum_1^\infty A_n\lambda^{-n}$. Any positive or zero powers of λ are excluded since $|R_\lambda| \to 1$ as $f \to 0$ $(\lambda \to \infty)$. The integral around the large semicircle is then $i\pi A_1$, with contributions for $n > 1$ vanishing as the radius of the semicircle tends to infinity.

We wish to use Cauchy's theorem around the contour. But singularities can arise within the contour if $R_\lambda(\lambda)$ is either zero or infinite. An infinity is excluded because the absorber is causal and passive. But the reflection coefficient can be zero, say at some wavelength λ_z. By the symmetry in the complex plane, there is another zero at $-\lambda_z^*$. These two zeros are removed by multiplying the reflection coefficient by a factor $(\lambda - \lambda_z^*)(\lambda + \lambda_z)/(\lambda - \lambda_z)(\lambda + \lambda_z^*)$ for each such zero point λ_z. This factor moves the two zeros into the lower half of the complex plane, outside the contour. The factor has unit magnitude for wavelengths along the real axis, and so does not modify the real-axis integral. It does introduce an extra term in the integral around the large semicircle. Having done this, Cauchy's theorem gives:

$$\int_0^\infty \ln[1/|R_\lambda(\lambda)|]\, d\lambda = -i\pi A_1/2 - 4\pi \sum \operatorname{Im} \lambda_z\,. \tag{2}$$

Since λ_z is in the upper half-plane, the last term is always negative. Then we have:

$$\int_0^\infty \ln[1/|R_\lambda(\lambda)|]\, d\lambda \leq -i\pi A_1/2\,. \tag{3}$$

Since the left-hand side is clearly real and positive, this equation only makes sense if A_1 is positive imaginary. We have obtained an upper limit on an integral of the reflection coefficient, which depends only on the first term of its low-frequency expansion.

3. Application to Chiral Materials

The reflection coefficient at normal incidence of a single-layer absorber can be written:

$$R = \frac{iZ_1 \tan(k_1 h) - Z_0}{iZ_1 \tan(k_1 h) + Z_0}\,. \tag{4}$$

Here Z_0 is the impedance of free space, Z_1 is the impedance in the medium, and k_1 is the wavenumber in the medium. With low-frequency approximations, this becomes $R \approx -1 + 4\pi i\mu_1 h/\lambda$. This gives the value of A_1 in the expansion, and leads to the limit quoted by Rozanov:

$$\int_0^\infty \ln[1/|R_\lambda(\lambda)|]\, d\lambda \leq 2\pi^2 \mu_1 h\,. \tag{5}$$

He also remarks that for a narrow-band absorber the factor of $2\pi^2$ should be replaced by 16. If this is done, and the variable changed from λ to f, we recover our limitation above. In a wavelength integral, the absorption bands of a Salisbury screen have decreasing width with a finite sum (figure 3), and they account for the difference between 16 and $2\pi^2$.

The calculation can be generalised to a multi-layer absorber, using a standard recursive process of computing the upward and downward going waves in each layer, starting at the back with the zero electric field boundary condition, and applying suitable propagation factors through each layer and field continuity conditions at each interface. At each stage one makes low-frequency approximations, keeping only the first order term. The algebra is lengthy, and the result is the same as for a single layer, except that we have a sum over the layers $\sum \mu_i h_i$ instead of a single term.

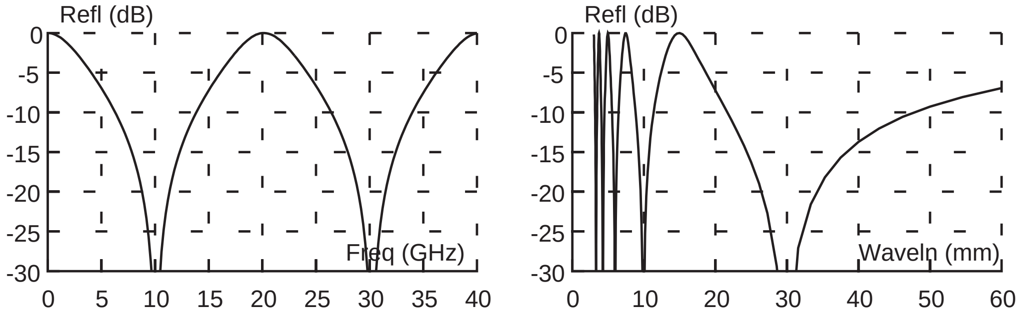

Figure 3: Reflection from single-layer absorber
((a) Function of frequency, (b) function of wavelength)

Turning to chiral materials, the details depend on the formalism used. If we use the form generally used by Lindell and Sihvola, we find that the reflection is given by equation 4, and the chirality does not appear [7]. Thus we immediately obtain the same value of A_1, and the same limit, as for a non-chiral medium. If we use the Post-Jaggard or the Lakhtakia-Varadan formalism, we find that the chirality only enters in the second order in frequency. Again, it does not affect the value of the first-order coefficient A_1, and we arrive at the same limit.

The omega medium (e.g. [8]) contains wires shaped like a Greek letter Ω, and provides a different electric-magnetic coupling than the usual chiral effect. However, taking the formulae for the reflection coefficient, and making low-frequency approximations, we find that that coupling parameter only enters in the second order in frequency. Again, it does not affect the value of the first-order coefficient A_1, and we arrive at the same limit.

4. Conclusions

We have shown that there is a fundamental limit on the broadband performance of a planar absorbing structure on a metal back, proportional to the total thickness and the permeability only. The same limit is found for chiral and omega materials as for purely dielectric materials.

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