Polarized Spatial Soliton in a Chiral Optical Fiber

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Abstract

The problem of soliton propagation in nonlinearity Kerr medium with linear optical activity and cubic anisotropy is considered. It is shown that the balance between the nonlinearity and linear girotropic results in the existence of spatial polarized solitons with fixed states of polarization. The chirality effect is characterized through the Born-Fedorov formalism and the results show modifications of the attenuation and nonlinear coefficient compared with the tipical coefficients in a nonlinear Schrödinger equation for a normal fiber in a regime of 1,55 and 1,3 μm.

1. Introduction

Chirality was firstly observed as optical activity and it corresponds to the rotation of the polarization plane, in a linear isotropic material. Phenomenonlogical studies establishes that the polarization plane rotation may be predicted by Maxwell's equations adding to the polarization P an additional term proportional to $\nabla \times \vec{E}$. The Drude-Born-Fedorov equations by satisfying the edge conditions[1], allows us to characterize the nonlinear chiral media through by the equations $D = \epsilon_n \vec{E} + \epsilon\zeta\nabla \times \vec{E}$ and $B = \mu_0(\vec{H} + \zeta\nabla \times \vec{H})$, where ϵ_n is the permittivity and ζ is the chiral coefficient. The pseudo scalar ζ represent the measure of chilarity and it has length units. [2]-[3]. It should also be considered the non local character of theses equations, since the polarization P (magnetization M) depent not only of E (H) but also of the rotor of E (rotor of H). Even though from an electromagnetic point of view a homogeneous chiral material may be discrete by different specific equations [3], in this work we will use the Drude-Born-Fedorov equations in optical fiber since they are the most adequate for the applications of our interest.

2. Basic Propagation Equation

Using equations the above equations, the corresponding Maxwell's equations are

$$\nabla \times \vec{H} = \frac{\partial\left(\epsilon_n \vec{E}\right)}{\partial t} + \sigma\vec{E} + \frac{\partial}{\partial t}\epsilon\zeta\left(\nabla \times \vec{E}\right) = \frac{\epsilon_n \partial\vec{E}}{\partial t} + \sigma\vec{E} + \epsilon\zeta\nabla \times \frac{\partial \vec{E}}{\partial t} \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0\frac{\partial \vec{H}}{\partial t} - \mu_0\zeta\frac{\partial\left(\nabla \times \vec{H}\right)}{\partial t} \tag{2}$$

If we make the follow considerations:

- The chiral media has a Kerr type non-linearity characterized by the refraction index such that the permittivity is $\epsilon_n = \epsilon_o + \epsilon_2\left|\vec{E}\right|^2$ [4], were ϵ_o is the lineal part and ϵ_2 is the non linear part, respectively, of ϵ_n.

- That the optical electric field E represent a located wave propagating in the direction

$$\vec{E}\left(\vec{r},t\right)=\left(\hat{x}+j\hat{y}\right)\Psi\left(\vec{r},t\right)e^{-j(kz-\omega_0 t)}=\vec{\Psi}e^{-j(kz-\omega_0 t)} \tag{3}$$

were $\vec{\Psi}$ represents the complex envelop.

- That the condition of a slowly variant envelop conditions its may be

$$\left|\frac{\partial^2\vec{\Psi}}{\partial z^2}\right| \ll \left|j2k\frac{\partial\vec{\Psi}}{\partial z}\right|;\left|\frac{\partial\vec{\Psi}}{\partial t}\right| \ll \left|j\omega_0\vec{\Psi}\right|;\left|\frac{\partial^2\left|\vec{\Psi}\right|^2\vec{\Psi}}{\partial t^2}\right| \ll \left|j\omega_0\frac{\partial\left|\vec{\Psi}\right|^2\vec{\Psi}}{\partial t}\right| \ll \left|j\omega_0\left|\vec{\Psi}\right|^2\vec{\Psi}\right| \tag{4}$$

- That the phenomenon of dispersion is included in heuristic form through the relation $\Delta k=\frac{1}{v}\frac{\partial}{\partial t}=\frac{\partial k}{\partial\omega}\frac{\partial}{\partial t}-j\frac{1}{2}\frac{\partial^2 k}{\partial\omega_0^2}\frac{\partial^2}{\partial t^2}-j\frac{1}{6}\frac{\partial^3 k}{\partial\omega_0^3}\frac{\partial^3}{\partial t^3}=\frac{k_0}{\omega_0}\frac{\partial}{\partial t}$

we obtain the following wave equation

$$j\left(\frac{\partial\phi}{\partial z^*}+\frac{1}{v_g}\frac{\partial\phi}{\partial t}\right)+\frac{1}{2}k''\frac{\partial^2\phi}{\partial t^2}-j\frac{1}{6}k'''\frac{\partial^3\phi}{\partial t^3}+\left(1-\zeta k_0\right)\left[\frac{j\omega\alpha}{2k_0}\phi-\frac{\beta\omega_0^2}{(2k_0)^3}\left|\phi\right|^2\phi\right]+\left(1-\frac{\zeta k_0}{2}\right)\zeta k_0^2\phi=0 \tag{5}$$

where $k'=\frac{\partial k}{\partial\omega}=\frac{1}{v_g}$; $k''=\frac{\partial^2 k}{\partial\omega^2}$; $k'''=\frac{\partial^3 k}{\partial\omega^3}$. Equation (5) describes the propagation of pulses in a chiral dispersive and nonlinear optical fiber. The analysis of each term is has follows [4]: The first term represent the evolution of pulse with distance; The second, third and fourth terms represent the dispersion of the optical fiber $k'(=1/v_g)$ and k'' correspond to the chromatic dispersion; k' indicates that the pulses moving which the group velocity, while that the dispersion of the group velocity (GVD) is represented by k'', which alters the relative phases of the frequency components of pulses producing its temporal widening. k'' is null in the region of 1.3 μm, For values of λ less than 1.3 μm, k'' is positive (normal dispersion region) and for values higher than 1.3 μm, is negative (anomalous dispersion region). k''' represent the slope of the group velocity dispersion, also denominated cubic dispersion and correspond to a higher order dispersion; important in ultra short pulses and in the second optical window where k'' is null (1.3 μm region). The cubic dispersion, besides, is important in fiber with shifted dispersion to the region of 1.5 μm. The fifth term is associated with the attenuation of the fiber (α), in this case those losses are weighed by the chirality of the fiber. $\left|\phi\right|^2\phi$ represent the nonlinear effect, and are due to the Kerr effect, which is characterized by having a refraction index depending on the intensity of the applied field. An index of this type for the case of optical fiber, means that there is a phase shift depending on the intensity and since the temporal changes of phase are also temporal changes of frequency, Its have that the Kerr type non linearity may alter and widening frequency spectrum of the pulse. This term also depends on the chirality of the fiber. The last term is highly associated to the chirality of the fiber.

3. Nonlinear Schrödinger Equation

In order to ease up the solution of the propagation equation the following changes of variables is introduced: $t'=t-\frac{z^*}{v_g}$ and $z'=z^*$, thus the original reference system will be $t=t'+\frac{z^*}{v_g}$ and $z^*=z'$ the equation (5) takes the form

$$j\frac{\partial\phi}{\partial z'}+\frac{1}{2}k''\frac{\partial^2\phi}{\partial t'^2}-j\frac{1}{6}k'''\frac{\partial^3\phi}{\partial t'^3}+j\frac{\alpha\omega_0}{2k_0}\left(1-\zeta k_0\right)\phi-\frac{\beta\omega_0^2}{\left(2k_0\right)^3}\left(1-\zeta k_0\right)\left|\phi\right|^2\phi+\zeta k_0^2\left(1-\frac{\zeta k_0}{2}\right)\phi=0 \tag{6}$$

Defining the new variables

$$q = \frac{\omega_0}{2k_0}\beta^{\frac{1}{3}}\phi, \quad \xi = \frac{\beta^{\frac{1}{3}}}{2k_0}z', \quad \tau = \sqrt{\frac{1}{2k_0k''}}\beta^{\frac{1}{6}}t', \quad \partial\tau^2 = \frac{\beta^{\frac{1}{3}}}{2k_0k''}\partial t'^2, \quad \partial\tau^3 = \frac{\beta^{\frac{1}{2}}}{(2k_0k'')^{3/2}}\partial t'^3$$

$$\gamma = \frac{\beta^{\frac{1}{6}}k'''}{6k''}\frac{1}{\sqrt{2k_0k''}}, \quad C = 1 - \zeta k_0, \ \Gamma = \frac{\omega\alpha}{\beta^{1/3}}$$

and operating algebraically we get the non linear Schrödinger equation for a chiral optical fiber.

$$j\frac{\partial q}{\partial \xi} + \frac{1}{2}\frac{\partial^2 q}{\partial \tau^2} - i\gamma\frac{\partial^3 q}{\partial \tau^3} + j\Gamma C q - C\left|q\right|^2 q + \frac{k_0}{\beta^{1/3}}\left(1 - C^2\right)q = 0 \tag{7}$$

4. Analysis of Results

The equation (7) represents the basic modeling of the pulse propagations in a chiral optical fiber dispersive and nonlinear. This is applicable both in the second and third optical windows. For the numerical calculation we use $k'' = -17,4$ ps^2/km, $\gamma = 0$, $\Gamma = 0$, which correspond to the anomalous region for a fiber length equal to 2.9 km . Fig. 1 and Fig. 2 correspond to one-order soliton with input power peak $P_0 = 0,87$ W and C=0,85 and 1,15 respectively. Fig. 2 shows an increasse of the intensity when the pulse propagates. This effect appears when ζk_0 is negative so if the losses (Γ) are included the chirality factor can compensate the typical decrease of the power pulse of the normal optical fiber. Fig. 3 and Fig. 4 correspond to the second-order solitons. Here we put $P_0 = 3,49$ W, this peak power is required to sopport the second order soliton. If we compare Fig. 3 and Fig. 4 , we see that with ζk_0 positive the signal is less distorted. Finally, Figs. 5 and 6 shows the behavior of the third-order solitons, $P_0 = 7,86$ W.

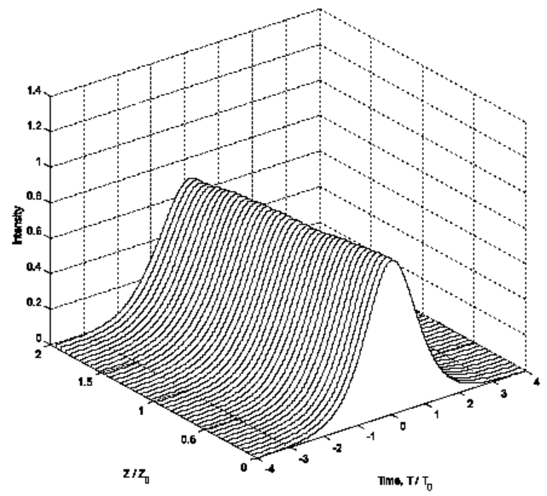

Fig. 1

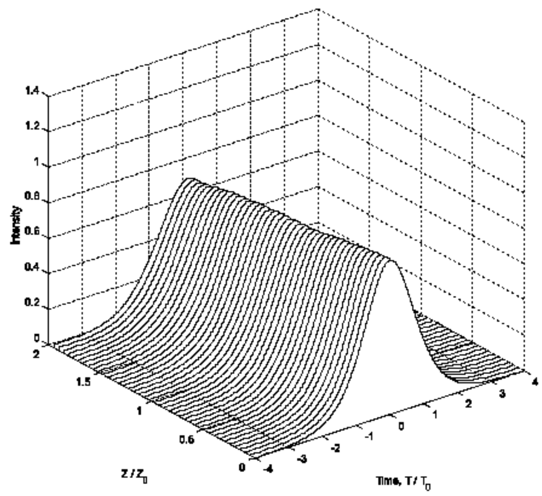

Fig. 2

5. Conclusion

In this work we have obtain the nonlinear Schrödinger equation for an optical fiber whose core is chiral dispersive and have nonlinear behavior. The effect of chirality is shown over the term associate to fiber lossy and to the nonlinear coefficient. The phenomenons that produce the dispersive effect and nonlinear in a non-chiral optical fiber (which produce the soliton propagation for example), are affected in case of using a chiral fiber, since to produce the same effect it will be necessary to operate the fiber in the normal dispersion regimen. The most important result in our work it the possibility to use the chirality of the fiber to cancel out losses and non linearitys of the optic fiber, which would allow to modify radically their behavior as channel of communications.

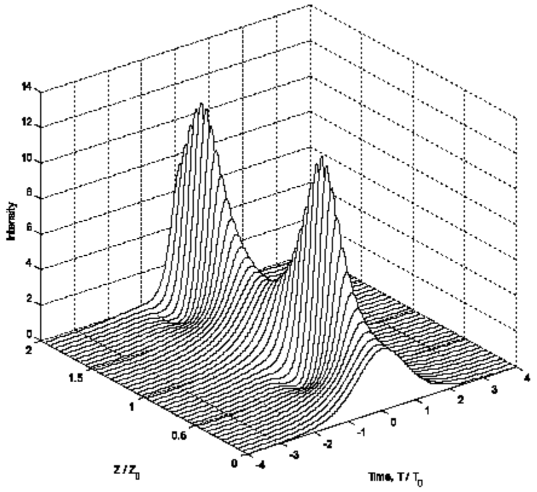

Fig. 3

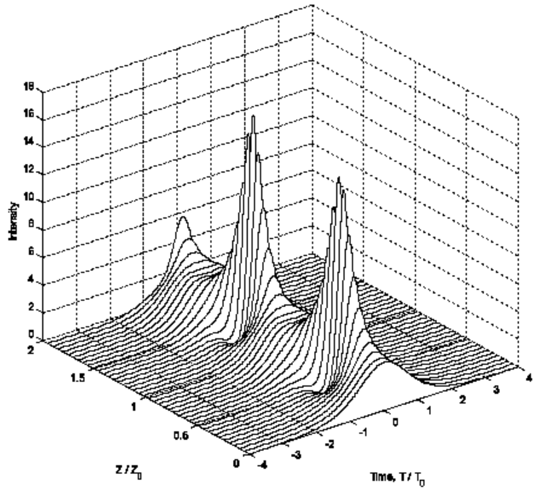

Fig. 4

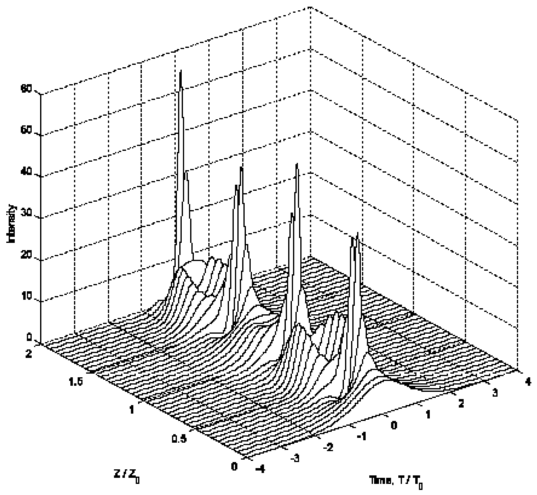

Fig. 5

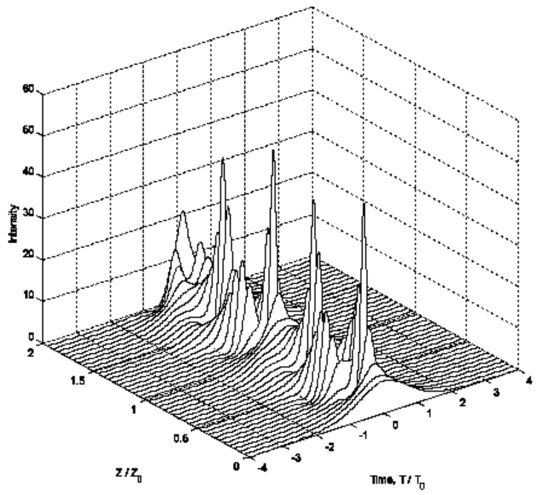

Fig. 6

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References

[1] A. Lakhtakia, V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media.* Lecture Notes Physics, 335, Springer-Verlag, 1985.
[2] H. Torres–Silva, P. H. Sakanaka, N. Reggiani, and C. Villarroel, "Electromagnetic properties of a chiral-plasma," *Pramana Journal of Physics*, 48, No. 1, 1997.
[3] H. Torres–Silva, "Chiro-plasma surface wave," *Advances in Complex Electromagnetic Materials.* Kruwer Academic Publishers, 38, Netherlands, 249, 1997.
[4] G. Agrawal, *Nonlinear Fiber Optics.* Academic: New York, 1995.
[5] M. Zamorano, H. Torres y C. Villarroel, "Ecuación de Schrödinger para una fibra óptica quiral," *R. Mexicana de Fisica*, 46(1),62, 2000.