Ubi Materia, Ibi Geometria

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Abstract

This invited presentation discusses the connection between geometry and matter. The theme of the paper, ubi materia, ibi geometria, is illuminated from the point of view of history of post-Renaissance science and also in terms of modern bi-anisotropic electromagnetics research. The citation is pinpointed in Johannes Kepler's philosophical texts, and an attempt is made to understand its meaning. The validity of 'ubi materia, ibi geometria' for the electromagnetic bi-anisotropics research is reconfirmed and a modified interpretation is proposed for it.

1. Preliminaries

Johannes Kepler declared 'Ubi materia, ibi geometria'. It is not difficult to imagine that a modern researcher in the field of chiral and bi-anisotropic electromagnetics concurs with this statement; 'where there is matter, there is geometry'. Matter and geometry, Nature and mathematics are intimately connected.

Ubi materia, ibi geometria. Citations to this formula appears often in scientific literature, sometimes with a reference to Kepler, sometimes just as it reads.[1] But it is difficult to find an exact citation from where and in what context in Kepler's writings this statement can be found. It seems to be well hidden; the only[2] place I have seen it is in the small tractate [6] *De fundamentis astrologiae certioribus*, shown with the surrounding text in Figure 1.[3] Where there is matter, there is geometry.

The observation of the connectedness of nature and mathematics was essential with the scientific revolution in the 16th and 17th centuries. In today's world where apparently nothing else than change is permanent, knowledge is said to be short-lived, and postmodernism is claimed to have replaced modernity.[4] But is it really so? Is Millennium the opening to a New Age? I would hesitate to agree. It is one of the aims of the present paper to defend Kepler's ethos which pronounces the reality of geometry in matter. If we are now going towards a new age, it should and could rather be a Renaissance of modernity.

[1] *Ubi materia, ibi geometria* can be found as a motto for chapters in mathematics books (for example, [1]), in general studies of the birth of new physics and astronomy, and as contribution to obscure rhetoric in speculative quantum physics texts. In fact, not only mathematicians and physicists use this slogan. It can be found in other environments, like of course philosophy and history, but also surveying [2], linguistics [3], and even jurisprudence [4]. A search in the world-wide-web with the string <ubi materia, ibi geometria> returns several virtual sites in rather unexpected places, most of them confusing. In the hunt for the origins of the title of the present paper, more interesting hints can be found using the digitally stored scholarly journal archive JSTOR [5].

[2] With this I do not imply that Kepler may not have written it also elsewhere in his great books (e.g., *Mysterium Cosmographicum, Astronomia Nova, Harmonices Mundi*).

[3] See footnote 12 on page 400 of [7].

[4] Physicists at large, of course, tend to have another opinion; see, for example, the delightful [8].

2. Matter and Geometry: Kepler's Platonism

Johannes Kepler (1571–1630) is rightfully seen as a central figure in the birth of the new physics. It is often said that the Copernican revolution[5] marks the end of Medieval times, and there indeed, Kepler's contribution was very essential in giving support to heliocentric cosmology and replacing the Aristotelian concepts in mechanics by new ones. For Kepler and Galileo, not to mention Newton, mathematics had to be included in the correct description of natural phenomena.

But in the case of Kepler the importance of mathematics was not only in arithmetic. Surely he was able to calculate tolerable approximations for the positions of planets. However, Kepler was in the sense Platonist that he believed in the rigorous and perfect structure of the universe. His celestial world had—at least in his early writings—a geometrically exact, nearly Pythagorean form, "his God [was] a geometer, and not an aritmetician" [7, p. 139]. This view is reflected in his models for the sizes of the planetary spheres that had harmonic ratios.[6] Kepler made ample use of various polyhedric volumes and it can be seen in the illustrations of his books, cf., e.g. Figure 2. The young Kepler was advised by Tycho Brahe to abandon his aprioristic speculations for more fruitful observational work. The incompatibility between the two approaches, the empiristic Tychonian on one hand, and geometrically ideal on the other, caused probably certain tension for Kepler during his later studies.

> ***Primam contrarietatem Aristoteles in metaphysicis recipit illam, quae est inter idem et aliud: volens supra geometriam altius et generalius philosophari. Mihi alteritas in creatis nulla aliunde esse videtur, quam ex materia aut occasione materiae, aut ubi materia, ibi geometria. Itaque quam Aristoteles dixit primam contrarietatem sine medio inter idem et aliud, eam ego in geometricis, philosophice consideratis, invenio esse primam quidem contrarietatem, sed cum medio, sic quidem, ut quod Aristoteli fuit aliud, unus terminus, eum nos plus et minus, duos terminos dirimamus.***

Figure 1: Part of Johannes Kepler's Thesis xx from *De fundamentis astrologiae certioribus*; (Opera Omnia, Vol. 1, p. 423). What is Kepler's intention in the second sentence?

Aside from astronomical and astrological theories, also in his Earth-bound studies Kepler was seeking causes for the form of matter. There he can be seen to follow more faithfully the metaphysics of Aristoteles.[7] For example, in his study of the snowflake (*Strena seu de Nive sexangula* ([10]),[8] Kepler searches the reason and cause for the sixfold symmetry, obvious in snow crystals (see, for example, Figure 3). His conclusion is that the cause for of the six-sided shape of a snowflake is a formative faculty already present in water in the liquid state and in vapour.

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[5] The term 'revolution' may have overtones that pertain to certain schools of the philosophy of science; by this choice of words I am not attaching myself to any of these, nor criticising them. The reader fluent in Finnish finds the discussion in [9] enlightening in respect how literally the metaphor of revolution can be taken in connection of Copernicanism.

[6] Kepler saw that there had to be six planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn) because only then the five regular polyhedra can be circumscribed between their spheres. In the earlier Ptolemaic system, there was place for seven planets (Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn); hence the Copernican system has to be valid (*Mysterium Cosmographicum*). (One might, however, with justification doubt that the system presented by Copernicus in his book *De Revolutionibus Orbium Coelestium* was simpler or aesthetically more pleasing than the theory in Ptolemy's *Almagest.*)

[7] According to the classical Aristotelian analysis, there are four causes for entities (things): material, formal, efficient, and final.

[8] See also [11, 12].

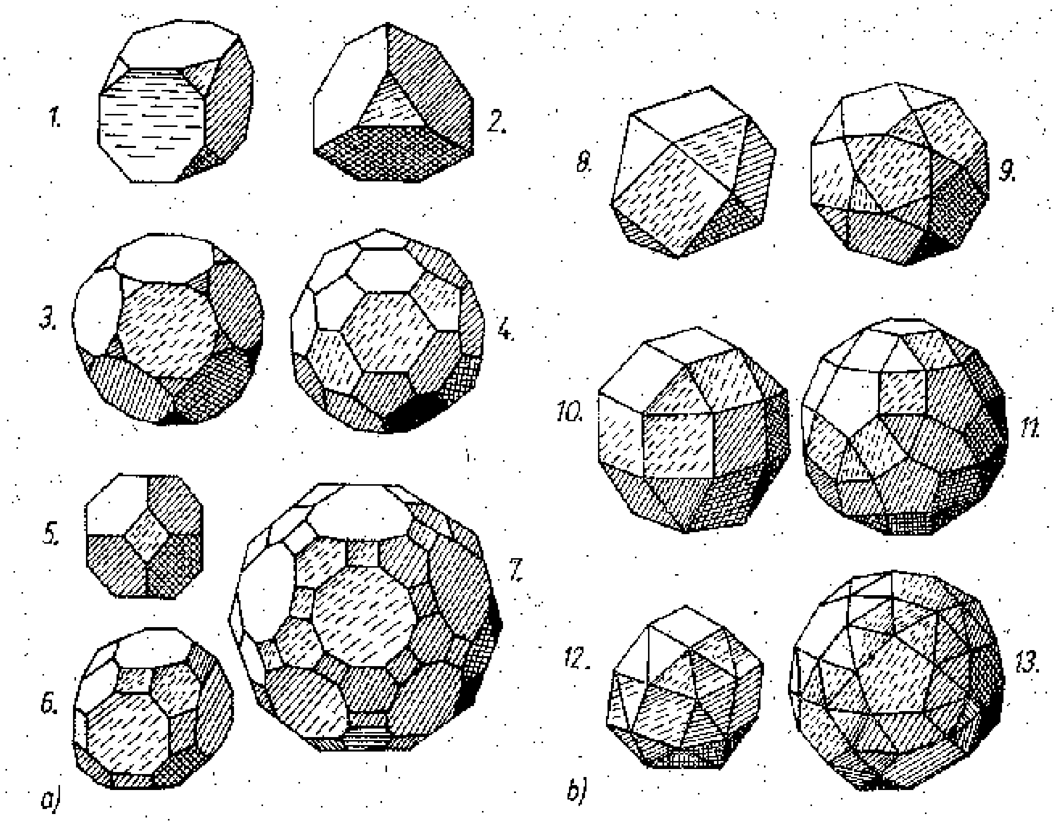

Figure 2: Archimedean polyhedra, used by Kepler in *Harmonice Mundi*.

What, then, did Kepler mean by his *Ubi materia, ibi geometria*? The citation, Figure 1 — *Mihi alteritas in creatis nulla aliunde esse videtur, quam ex materia aut occasione materiae, aut ubi materia, ibi geometria* — is not perfectly apparent. Somehow Kepler is saying that matter, and its emergence (*occasione*) is the cause of a certain property in created objects: *alteritas*. 'Alteritas' might be 'variety' or 'otherness', and this is an essential concept which in some sense in Kepler's mind gets connected with geometry, as the conclusion of the sentence ('in other words, where there is matter, there is geometry') shows. One reasonable interpretation might be just to guess that Kepler emphasises his idealistic belief in perfect shapes immanent within all matter.[9] It is probably not too bold to presume that Kepler would have loved today's solid-state physics classes, and would have much preferred them over semiconductor engineering textbooks where impurities and defects ruin the clean symmetries in solid matter.

3. Geometry and Matter: Bi-Anisotropics in the 21th Century

Ubi materia, ibi geometria. These words can a materials scientist safely speak out in the *Bianisotropics 2000* meeting in Lisbon. To calculate the effective dielectric and magnetic medium parameters of simple or complex materials, knowledge about their geometrical structure has to be made use of. The macroscopic properties of solid materials are connected to their microscopic crystallography. Kepler's maxim has even been reworded more strongly in the form of Neumann's principle: No asymmetry may be exhibited by any property of the crystal which is not possessed by the crystal itself [15]. In other words, the special characteristics of the geometrical constellation of solid matter "shine through" in the properties of its measurable properties.

[9] I welcome competing interpretations!

Pierre Curie has expressed the principle in the form 'C'est la dissymétrie, qui creé le phénomène' (It is asymmetry which creates the phenomenon).

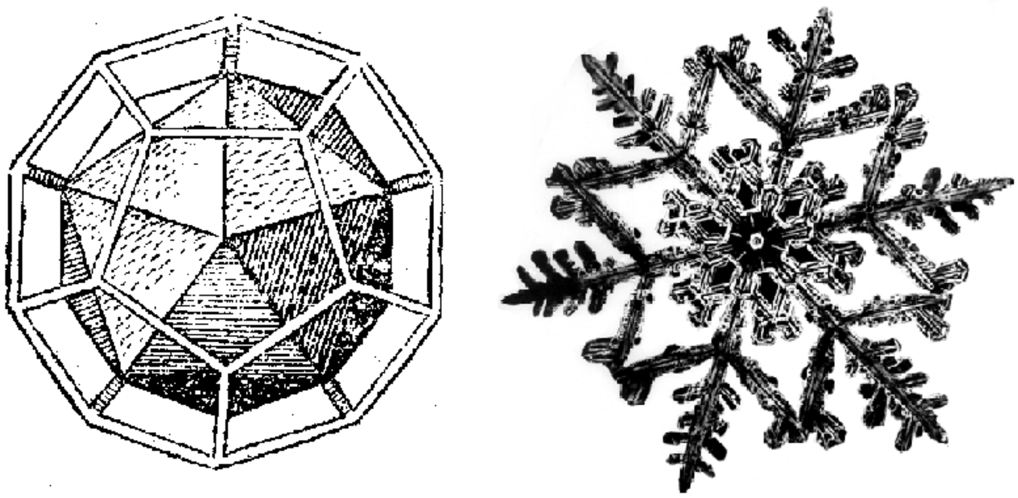

Figure 3: The togetherness of matter and geometry was obvious for Kepler also in his studies of snow [10]. On the left, one of his favourites, icosahedron in Platonic dodecahedron. To the right side, one of Wilson A. Bentley's snowflake photographs. (Spending decades during the early last century, W.A. Bentley of Jericho, Vermont (New England), patiently and skillfully took thousands of photographic images of snow crystals. These can be admired in the Dover edition [13] of his book and nowadays also on a CD-rom [14].) Note the sixfold symmetry in the snowflake.

Indeed, where a physicist or an electrical engineer needs to find out *ab initio* the effective properties of matter, the presence or absence of symmetry, that is geometry, has to be dealt with. But could Kepler's watchword be approached from another direction? What about understanding it in a way such that the role of geometry is stressed more; so that geometry begets matter? In such a reading of the principle, we in some sense surpass the Neumann–Curie interpretation. The properties of matter are determined by its geometrical description; this is then true but trivial: what is essential is that these properties can be very varied and unexpected. Why? We only need to think about engineering applications such as chirality, photonic band gap structures, or liquid crystals.

More technical support to such an interpretation for the interplay between geometry and matter is provided by the basic material relations of bi-anisotropic electromagnetics:

$$\mathbf{D} = \epsilon \bullet \mathbf{E} + \xi \bullet \mathbf{H} \tag{1}$$

$$\mathbf{B} = \zeta \bullet \mathbf{E} + \mu \bullet \mathbf{H} \tag{2}$$

The field vectors are denoted here by $\mathbf{E}$ (electric) and $\mathbf{H}$ (magnetic), and the responses in this representation are the flux densities $\mathbf{D}$ (electric) and $\mathbf{B}$ (magnetic). The relations between these vectors are given by the material parameters, permittivity ϵ, permeability μ, and the two magnetoelectric cross terms ξ and ζ.[10] The geometry of matter determines the character of the

[10] I am not discussing here the question which one of the magnetic fields ($\mathbf{H}$ or $\mathbf{B}$) is primary and should appear on the right-hand side of the constitutive relations (to read a sharp-worded argument against the above use, look [16]). Also, I am well aware of the fact that one has to be careful with the order of the magnitude and balance of electric and magnetic polarisation terms on the right-hand side of these relations (see, for example, [17]). In terms of the focus of the paper, these points are not crucial; also, the form of the relations (1)–(2) is familiar and much-used in Bianisotropics meetings [18].

quantities ϵ, μ, ξ, and ζ, and the nature of the product marked by $\bullet$ in (1)–(2). For the simplest isotropic case, the product is a plain multiplication by a scalar. The other extreme is that all four medium "parameters" are full dyadics (alternatively, tensors of second rank) and then the number of degrees of freedom in the bi-anisotropic description of such matter is $4 \times 9 = 36$.[11]

If the medium is anisotropic, its material dyadics, for example the permittivity $\overline{\overline{\epsilon}}$, have a structure which reflects faithfully the internal geometry of the matter, according to the Neumann principle. But how do the magnetoelectric effects arise? Obviously through geometry.

Chirality is one reciprocal type of these magnetoelectric effects. Chirality can be said to be present in materials that have a handed microstructure. If the medium is predominantly right- or left-handed, macroscopic effects of chirality can be observed. One observable optical or electromagnetic effect of chiral media is their ability to rotate the polarisation plane of the incident wave, and on the level of dipole moments in the medium the effect is electrically caused magnetic polarisation density and vice versa. But these effects take place because of the left–right symmetry is broken.[12]

Another affirmation for the strong interpretation of the Kepler principle is artificial magnetism. 'Natural' magnetism is present in certain materials, like iron, and in electromagnetics applications this property is taken into account by the $B - H$ relation. The origin of the magnetic properties is not important. It is not easy to talk about the geometrical cause of magnetic permeability in quantum mechanics. But because macroscopic electric loops act as magnetic moments, it is possible to synthetise a medium with magnetic permeability. Also in modelling of chiral, bi-anisotropic, and other complex materials, the observation of the appearance of magnetic properties for materials with non-magnetic constituents has been emphasised [21, 22, 23]. If interested in "artificial" magnetism in natural media, see [24] for the diamagnetic effect of wet snow due to the funicularly circling water phase in the ice matrix.

Finally, it is perhaps proper to remind of one important electromagnetics application that hinges on the geometry–matter interaction. This is the field with many names: photonic band gaps, electromagnetic or photonic superlattices, photonic crystals, periodic quasicrystal structures, etc. [25]. There, the geometry of the structure is built in such a way that from the electromagnetic point of view the medium looks transparent or opaque depending on the wavelength, thus giving access to a whole new range of material properties. With such prospects in sight, it is not overoptimistic to foresee a revival for Kepler's credo. *Ubi materia, ibi geometria.*

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[11] This number is outrageously high. Brave attempts to control its freedom have found their way into the electromagnetics literature [19].

[12] In the subatomic level, physicists use the term 'parity' in connection of chirality. In the weak interaction process, parity is broken [20].

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