Electromagnetic Waves in Chiral Media with Compensated Anisotropy

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Abstract

We consider and compare three different cases of anisotropy compensation in chiral media. The first case concerns natural crystals or nonmagnetic superlattices, the second case is the class of stratified-periodic structures, which are homogeneous media in the long-wave approximation and have simultaneously dielectric and magnetic properties. The third possible case can be realized as inhomogeneous structures with spiral anisotropy of dielectric and magnetic properties.

1. Natural Crystals and Nonmagnetic Superlattices

Effect of compensation of dielectric anisotropy of natural gyrotropic crystals $AgGaS_2$ (silver gallium sulphide) and $CdGa_2S_4$ (cadmium gallium sulphide) was observed for the first time by M.V. Hobden in 1967 and 1968 [1, 2]. For these crystals the dispersion curves of the main values of the uniaxial tensor of permittivity $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ cross each other at a certain frequency. For the light of this frequency crystal becomes isotropic and its chiral properties are not masked by linear birefringence. The crystal $AgGaS_2$ (class $\overline{4}2m$) is optically isotropic at the wavelength $\lambda_0 = 4970$ Ångström, the rotatory power is equal to 522 degrees/mm [1]. The crystal $CdGa_2S_4$ of class $\overline{4}$ has the isotropic point at the wavelength $\lambda_0 = 4872$ Ångström. This crystal shows the rotatory power of 17.3 degrees/mm and 11.6 degrees/mm for different directions of light propagation [2].

Effect of compensation of dielectric anisotropy is possible also in nonmagnetic stratified-periodic media (superlattices), where the "isotropic point" can be shifted under the action of external magnetic fields or under elastic deformation of crystal.

Media with new optical properties can be created by combination of crystals possessing necessary features. Modern technology allows to manufacture multilayered periodic systems (superlattices) with a period from 10 to 1000 Ångström [3]. Electromagnetic properties of superlattices are easily modelled in the long wavelength approximation, which is valid if the wavelength of electromagnetic or ultrasonic waves propagating in the lattice is large compared to the period of the structure. In this case it is possible to consider a superlattice as a uniform medium characterised by a set of effective parameters. Thereby, properties of superlattices can combine useful properties of their constituents, that is, crystals, from which these lattices are formed. The important property of superlattices is the difference between their crystallographic symmetry and the symmetry of crystals used as layers. For example, if layers are isotropic, superlattice as a whole is a uniaxial crystal [4, 5]. Due to such changing of symmetry, interesting effects are possible in superlattices, for instance, acousto-optical interactions [6, 7]. In the last years effective elastic moduli of superlattices with arbitrary crystallographic symmetry of the layers [3, 8], as well as elastooptical [3, 4], piezo-electric [9, 10], electro-optical [11] and nonlinear optical [12, 13] coefficients were theoretically estimated.

Alongside with dielectric, elastic, elasto-optic, piezo-electric and nonlinear properties of superlattices [3]–[13] also their chiral properties arise interest. In particular, it is possible to create superlattices which are dielectrically isotropic for the light at a certain frequency [14, 15].

The electromagnetic properties of chiral superlattices can be described by the constitutive relations [16, 17]

$$\mathbf{D} = \epsilon_0 \overline{\overline{\varepsilon}}^{\mathrm{eff}} \cdot \mathbf{E} - j\sqrt{\epsilon_0\mu_0}\,\overline{\overline{\kappa}}^{\mathrm{eff}} \cdot \mathbf{H}, \quad \mathbf{B} = \mu_0 \overline{\overline{\mu}}^{\mathrm{eff}} \cdot \mathbf{H} + j\sqrt{\epsilon_0\mu_0}\left(\overline{\overline{\kappa}}^{\mathrm{eff}}\right)^T \cdot \mathbf{E} \tag{1}$$

with effective tensors of permittivity $\overline{\overline{\varepsilon}}^{\mathrm{eff}}$, chirality $\overline{\overline{\kappa}}^{\mathrm{eff}}$, and permeability $\overline{\overline{\mu}}^{\mathrm{eff}}$. Here the values with index "eff" correspond to the effective medium, and the primed and nonprimed values correspond to the two layers which form the superlattice. The effect of compensation of dielectric anisotropy is possible in nonmagnetic superlattices ($\overline{\overline{\mu}} = \overline{\overline{\mu}}' = \overline{\overline{\mu}}^{\mathrm{eff}} = 1$) formed by two crystals with uniaxial symmetry, whose optical axes are orthogonal to the boundaries of the layers. The condition of isotropy of the effective permittivity of a multilayered periodic structure has the following form:

$$\varepsilon'_{11} + x\left(\varepsilon_{11} - \varepsilon'_{11}\right) = \frac{\varepsilon'_{33}\varepsilon_{33}}{\varepsilon_{33} + x\left(\varepsilon'_{33} - \varepsilon_{33}\right)} \tag{2}$$

The axis number 3 is orthogonal to the boundaries of the layers. The period of the structure $\mathcal{D}$ is connected with the thicknesses of the layers by the relation $\mathcal{D} = d + d'$. The notations for the relative thickness of each layer have been introduced: $x = d/\mathcal{D}$, $1 - x = d'/\mathcal{D}$.

As the graphical method shows, equation (2) has a real root on the interval $0 < x < 1$, if the first layer is a positive crystal, but the second one is a negative crystal. This means that relation $\operatorname{sgn}\left(\varepsilon_{11} - \varepsilon_{33}\right) = -\operatorname{sgn}\left(\varepsilon'_{11} - \varepsilon'_{33}\right)$ holds. In this case mutual compensation of the anisotropy of the permittivity of layers takes place, and the properties of the superlattice with respect to the light of corresponding frequency are described by an effective scalar parameter of permittivity and by the effective tensor of chirality.

2. Superlattices with Dielectric and Magnetic Properties

Effect of mutual compensation of dielectric and magnetic anisotropy in nongyrotropic media was predicted by F.I. Fedorov [16]. This effect becomes possible in the medium if the tensors of its permittivity and permeability are proportional to each other at a certain frequency of light. This condition can be reached in periodic stratified structures, whose effective parameters combine dielectric and magnetic properties of the layers. In the long-wave approximation propagation of light in such media can be described by effective tensors of permittivity and permeability.

We have averaged the vectors of electromagnetic field in the volume of the crystal in accordance with the methods developed in paper [3], which allows to determine the effective parameters, characterising the optical properties of the chiral superlattice [14, 15]:

$$\frac{1}{\varepsilon_{33}}, \quad \frac{\varepsilon_{i3}}{\varepsilon_{33}}, \quad \varepsilon_{ik} - \frac{\varepsilon_{i3}\varepsilon_{k3}}{\varepsilon_{33}}, \quad \frac{1}{\mu_{33}}, \quad \frac{\mu_{i3}}{\mu_{33}}, \quad \mu_{ik} - \frac{\mu_{i3}\mu_{k3}}{\mu_{33}} \tag{3}$$

$$\frac{\kappa_{33}}{\varepsilon_{33}\mu_{33}}, \quad \frac{1}{\varepsilon_{33}}\left(\kappa_{3m} - \kappa_{33}\frac{\mu_{m3}}{\mu_{33}}\right), \quad \frac{1}{\mu_{33}}\left(\kappa_{m3} - \kappa_{33}\frac{\varepsilon_{m3}}{\varepsilon_{33}}\right) \tag{4}$$

$$\kappa_{mn} - \frac{\varepsilon_{m3}}{\varepsilon_{33}}\kappa_{3n} - \left(\kappa_{m3} - \kappa_{33}\frac{\varepsilon_{m3}}{\varepsilon_{33}}\right)\frac{\mu_{n3}}{\mu_{33}} \tag{5}$$

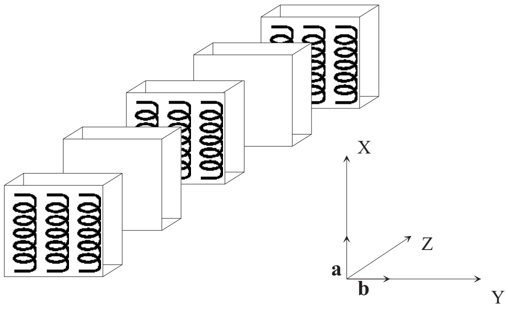

Figure 1: Geometry of the problem. The axes of the spirals are oriented along the x axis. The incident wave propagates along the z axis.

We have to substitute all these values in the generic relation

$$A^{\text{eff}} = xA + (1-x)\,A' \tag{6}$$

where indices i, k, m, n take values 1 and 2. By means of expressions (3) and relation (6) it is possible to determine the components of the effective tensor of permittivity and permeability, and by means of (4), (5), and (6) we can find the components of the effective tensor of chirality for a superlattice with arbitrary crystallographic symmetry of layers. Let us consider the case when a superlattice is formed by uniaxial crystals with the optical axes oriented perpendicularly to the boundaries of layers, i.e. along the z-axis (unit vector $\mathbf{z}_0$). The z-components are marked by index 3. The tensors of the permittivity and permeability of the first layer can be written as

$$\overline{\overline{\varepsilon}} = \varepsilon_{11}\,\overline{\overline{I}} + (\varepsilon_{33} - \varepsilon_{11})\,\mathbf{z}_0\mathbf{z}_0, \quad \overline{\overline{\mu}} = \mu_{11}\,\overline{\overline{I}} + (\mu_{33} - \mu_{11})\,\mathbf{z}_0\mathbf{z}_0 \tag{7}$$

where $\varepsilon_{11} = \varepsilon_{22}$, $\mu_{11} = \mu_{22}$. Tensors $\overline{\overline{\varepsilon}}'$ and $\overline{\overline{\mu}}'$ of the second layer have a similar type. In this case, as follows from relations (3) and (6), the superlattice is also a uniaxial crystal with the effective tensors of permittivity and permeability

$$\overline{\overline{\varepsilon}}^{\text{eff}} = \varepsilon_{11}^{\text{eff}}\,\overline{\overline{I}} + \left(\varepsilon_{33}^{\text{eff}} - \varepsilon_{11}^{\text{eff}}\right)\mathbf{z}_0\mathbf{z}_0, \quad \overline{\overline{\mu}}^{\text{eff}} = \mu_{11}^{\text{eff}}\,\overline{\overline{I}} + \left(\mu_{33}^{\text{eff}} - \mu_{11}^{\text{eff}}\right)\mathbf{z}_0\mathbf{z}_0 \tag{8}$$

where $\varepsilon_{11}^{\text{eff}} = \varepsilon_{22}^{\text{eff}}$, $\mu_{11}^{\text{eff}} = \mu_{22}^{\text{eff}}$. Mutual compensation of dielectric and magnetic anisotropies in the superlattice takes place at the condition

$$\frac{\varepsilon_{11}^{\text{eff}}}{\mu_{11}^{\text{eff}}} = \frac{\varepsilon_{33}^{\text{eff}}}{\mu_{33}^{\text{eff}}} \tag{9}$$

By means of relations (3) and (6), condition (9) can be presented in the following form:

$$\frac{x\varepsilon_{11} + (1-x)\,\varepsilon_{11}'}{x\mu_{11} + (1-x)\,\mu_{11}'} = \frac{\varepsilon_{33}\varepsilon_{33}'\left[x\mu_{33}' + (1-x)\,\mu_{33}\right]}{\mu_{33}\mu_{33}'\left[x\varepsilon_{33}' + (1-x)\,\varepsilon_{33}\right]} \tag{10}$$

Let us introduce notations for the mean values of the permittivity and permeability:

$$<\varepsilon> = \frac{1}{2}\left(\varepsilon_{11} + \varepsilon_{33}\right), \quad <\mu> = \frac{1}{2}\left(\mu_{11} + \mu_{33}\right) \tag{11}$$

and notations for the anisotropy of tensors $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$:

$$\Delta\varepsilon = \frac{1}{2}\left(\varepsilon_{11} - \varepsilon_{33}\right), \quad \Delta\mu = \frac{1}{2}\left(\mu_{11} - \mu_{33}\right) \tag{12}$$

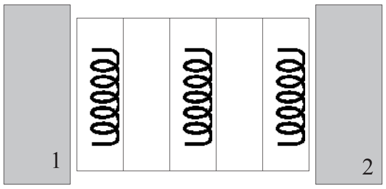

Figure 2: The scheme of the device. 1 is a polarizer, 2 is an analyzer.

Similar notations are used also for the primed and effective values. It is easy to prove (graphically) that equation (10) has a real root x on interval $0 < x < 1$, if the following conditions hold simultaneously:

$$\frac{\Delta\varepsilon'}{<\varepsilon'>} < \frac{\Delta\mu'}{<\mu'>}, \qquad \frac{\Delta\varepsilon}{<\varepsilon>} > \frac{\Delta\mu}{<\mu>} \tag{13}$$

Inequalities (13) mean that for the first layer the relative anisotropy of the permittivity exceeds the relative anisotropy of the permeability. For the other layer the inverse relation is true and the anisotropy of magnetic properties dominates. For instance, this case is possible if the first layer is a nonmagnetic crystal with parameters $\varepsilon_{11} > \varepsilon_{33}$, $\mu_{11} = \mu_{33} = 1$. At the same time the second layer is a crystal possessing only magnetic anisotropy: $\mu'_{11} > \mu'_{33}$, $\varepsilon'_{11} = \varepsilon'_{33} = 1$.

Using condition (9) in the first-order approximation on parameters $\Delta\varepsilon$ and $\Delta\mu$, we find the following relation:

$$\frac{\Delta\varepsilon^{\rm eff}}{<\varepsilon^{\rm eff}>} = \frac{\Delta\mu^{\rm eff}}{<\mu^{\rm eff}>} \tag{14}$$

This formula means that the anisotropy of dielectric properties compensates the anisotropy of magnetic properties. Because of the frequency dispersion of the parameters of the crystals the considered effect of mutual compensation of dielectric and magnetic anisotropies can exist for a certain light frequency only. Thus, superlattices with the specified properties can be used in devices with frequency-selective transmission of light.

Anisotropic chiral media with pronounced magnetic properties can be realized as a periodic superlattice (period $\mathcal{D}$) in which every period consists of two layers (see Figure 1). One layer is an array of parallel spirals, and this layer exhibits only anisotropic dielectric and chiral properties (manufacture of such media is described in [18]). The other layer has anisotropic magnetic properties (it can be a weakly magnetized ferrite such that off-diagonal components of the permeability dyadic are small, but diagonal components are electrically controlled, and losses can be neglected). The scheme of the device is presented in Figure 2 [19] for the case when optical axes of each layer and of superlattices as a whole are oriented parallelly to the boundaries of layers. Also in this case the effect of compensation of dielectric anisotropy is possible which is similar to the case considered above when the optical axes were oriented perpendicularly to the boundaries of layers.

Similar selective effects take place also in isotropic chiral slabs, since chirality parameter is frequency dependent. We see that due to anisotropy of permittivity and permeability, selectivity can be improved. Indeed, at the point of anisotropy compensation, difference between wavenumbers of two eigenmodes has a minimum. When the balance (14) is not fulfilled (due to changed frequency), this difference increases sharply. The thickness of the sample is chosen as

$$L = \frac{\pi}{2\theta} \tag{15}$$

where $\theta = k_0\kappa_{11}/2$ is the specific rotation of the polarization plane. The angle between the planes of the polarizer and analyser is $\pi/2$. On Figure 3 we demonstrate a typical dependence of the rotation angle on the effective permittivity. For the chosen parameter values the compensation

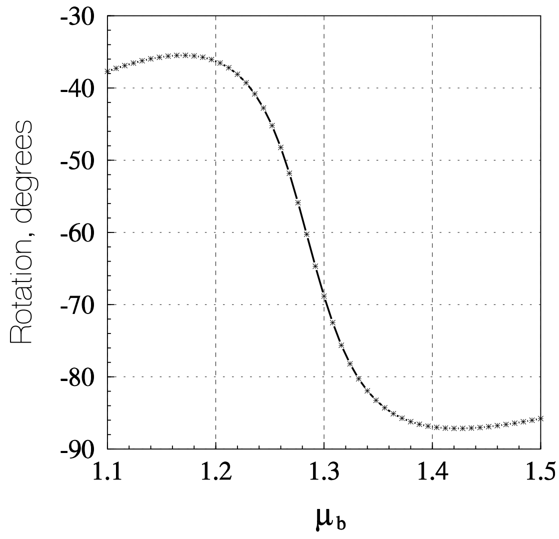

Figure 3: The rotation of the polarization ellipsis of the reflected wave as a function of $\mu_b = \mu_{22}^{\text{eff}}$. The incident wave polarization is perpendicular to the direction of the helix axes (along vector **b**), $\epsilon_{22}^{\text{eff}} = 3.9$, $\epsilon_{11}^{\text{eff}} = 3$, $\kappa_{11}^{\text{eff}} = 0.45$, $\mu_{11}^{\text{eff}} = 1$, $\omega/(2\pi) = 14$ GHz.

point (14) corresponds to $\mu_{22}^{\text{eff}} = 1.3$. We observe that close to this point the rotational power is indeed very sensitive to the material parameter values. This supports our expectation that the use of magnetically anisotropic structures can give a possibility to improve frequency selectivity of microwave and optical filters.

3. Spirally Inhomogeneous Media

Mutual compensation of dielectric and magnetic anisotropy is possible also in spirally inhomogeneous media. In the first case, investigated by Hobden, compensation of anisotropy leads to disappearance of linear birefringence of light. Now, in media with a spiral structure, disappearance of diffraction of waves becomes possible at the condition $\frac{\varepsilon_{11}}{\mu_{11}} = \frac{\varepsilon_{22}}{\mu_{22}}$ or $\frac{\Delta\varepsilon}{<\varepsilon>} = -\frac{\Delta\mu}{<\mu>}$ [20, 21]. In all three cases the properties of eigenwaves of media are considerably changed near the "isotropic point", which can be used for the design of devices for transformation of polarization of electromagnetic waves.

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