Eigen Waves of Periodic Layered Structure of Complex Arrays

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Abstract

The characteristics of eigen waves of periodical structures consist of 2-D arrays of strip particles, in particular having the shape of letters C, S and Ω are studied. Analytical and numerical results are presented. Study of eigen waves can be used for analyze property of polarization transformation by single complex array and multi-layered one.

1. Introduction

The layered periodic structure can be used as transformer of polarization of reflected and transmitted electromagnetic fields, absorber in the case a loss medium are placed between layers, and polarized and frequency selective surfaces. They has properties typical for photonic band-gap crystals. Analysis of characteristics of structure that has finite number of layers and semi infinite structure also can be carried by using the characteristics of eigen waves of infinite periodic structure [1], [2].

Let's consider an infinite layered periodic structure that is shown in the Fig. 1. Each layer represents a plane periodic on two directions array of strip particles. The particles have a complex shape, in particular, the shape of letters C, S and Ω. Mirror non-symmetric particles, such as S-shaped, are called planochiral [3]. All layers of structure are identical.

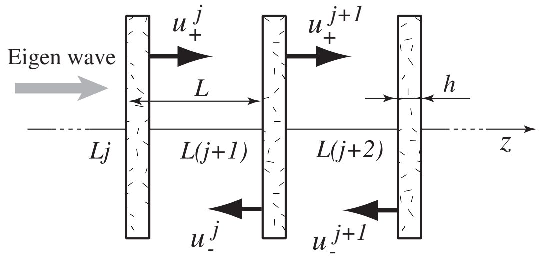

Figure 1: Infinite periodic layered structure of 2-D arrays

If the operators of reflection and transmission of a single layer are known it is easy to derive a homogeneous system of linear algebraic equations concerning vectors of amplitudes of plane waves that are propagated towards each other in every gap between layers. These form field of the eigen waves of an infinite structure. The condition of nonzero solution of a homogeneous system of equations is dispersing equation to which satisfy propagation constants of eigen waves. The distribution of electromagnetic field amplitude of an eigen wave and its polarization in gaps between layers can be derived from a solution of the system of equations.

2. Equations of Eigen Waves

For the sake of simplicity we shall confine to the most important case for applications. This case is one-wave scattering by single array on the assumption that both its periods are less than a wavelength. Under these conditions reflection and transmission operators of a single layer can be presented by the second order square matrices. The solution of the problem of electromagnetic wave scattering by a single array of strip particles of the complex shape is known [4], [5].

Electromagnetic field of eigen wave is the field of plane waves inside layered structure that propagated along axis Oz in positive and negative directions between the boundaries of neighbor layers. The field is represented in the j gap between layers as following

$$\vec{u}^j = \vec{u}^j_+ + \vec{u}^j_-, \qquad Lj + h < z < L(j+1) \tag{1}$$

where

$$\vec{u}^j_+ = \vec{A}^j_+ e^{ik(z-Lj-h)}, \qquad \vec{u}^j_- = \vec{A}^j_- e^{-ik(z-Lj-L)},$$

L is the structure period along Oz axis, h is the thickness of layer, it is assumed field time dependence $e^{-i\omega t}$. The wave amplitudes of eigen wave between neighbor gaps are transformed in accordance with formula

$$\vec{A}^{j+1}_\pm = e^{i\beta L}\vec{A}^j_\pm \tag{2}$$

where β is propagation constant of eigen wave. Wave amplitudes in a neighbor gaps are connected by two vector equations

$$\vec{A}^{j+1}_+ = t^- e^{ik\Delta}\vec{A}^j_+ + r^+ e^{ik\Delta}\vec{A}^{j+1}_- \tag{3}$$

$$\vec{A}^j_- = r^- e^{ik\Delta}\vec{A}^j_+ + t^+ e^{ik\Delta}\vec{A}^{j+1}_- \tag{4}$$

Here $\Delta = L - h$, $r^\pm$ and $t^\pm$ are reflection and transmission operators of single layer for the cases of incident of electromagnetic wave in positive "+" and negative "-" directions of axis Oz. After take into account the Floquet condition (2) we can rewrite these equations for amplitudes plane waves that forming eigen wave of structure in the gap between layers as following

$$(I - t^- e^{ik\Delta}e^{-i\beta L})\vec{A}^j_+ - r^+ e^{ik\Delta}\vec{A}^j_- = 0 \tag{5}$$

$$r^- e^{ik\Delta}\vec{A}^j_+ - (I - t^+ e^{ik\Delta}e^{i\beta L})\vec{A}^j_- = 0 \tag{6}$$

System of equations are invariant respect to change indexes "$\pm$" by "$\mp$" and change the sign of constant of propagation β of eigen wave simultaneously. This property is consequence of the invariant of structure properties respect to choosing of direction of axis Oz.

Let us assume that layers of structure have not non-reciprocal elements and as consequence the matrix of operator reflection and transmission are symmetrical. In this case and so as we take into account only propagated partial waves the operators t^+ and t^- are equal. If the layer is symmetric regard to its average plane the matrices of operators r^+ and r^- are equal also. We shall restrict for simplicity this case only. The system of equations regard to amplitudes of eigen wave in this case is follow

$$(I - \tilde{t}e^{-i\beta L})\vec{A}^j_+ - \tilde{r}\vec{A}^j_- = 0 \tag{7}$$

$$\tilde{r}\vec{A}^j_+ - (I - \tilde{t}e^{i\beta L})\vec{A}^j_- = 0 \tag{8}$$

where $\tilde{t} = te^{ik\Delta}$, $\tilde{r} = re^{ik\Delta}$.

3. Analytical Results

If non-diagonal elements of matrices $\tilde{r}$ and $\tilde{t}$ are equal to zero there are two independent systems of equations regard to amplitudes of two line polarized along axis Ox and axis Oy eigen waves. Dispersion equations have the form

$$2\tilde{t}_{xx}\cos\beta L = 1 + \tilde{t}^2_{xx} - \tilde{r}^2_{xx} \tag{9}$$

$$2\tilde{t}_{yy}\cos\beta L = 1 + \tilde{t}_{yy}^2 - \tilde{r}_{yy}^2 \tag{10}$$

Two eigen waves have different phase velocities in general case.

Let us now suggest that elements of reflection and transmission matrices are in accord with equations

$$\tilde{r}_{xx} = \tilde{r}_{yy}, \qquad \tilde{t}_{xx} = \tilde{t}_{yy}, \qquad \tilde{r}_{xy} = \tilde{t}_{xy}$$

Dispersion equations can be wrote in the form

$$2(\tilde{t}_{xx} + \tilde{r}_{xy})\cos\beta L = 1 + \tilde{t}_{xx}^2 - \tilde{r}_{xx}^2 - 2\tilde{r}_{xy}(\tilde{r}_{xx} - \tilde{t}_{xx}) \tag{11}$$

$$2(\tilde{t}_{xx} - \tilde{r}_{xy})\cos\beta L = 1 + \tilde{t}_{xx}^2 - \tilde{r}_{xx}^2 + 2\tilde{r}_{xy}(\tilde{r}_{xx} - \tilde{t}_{xx}) \tag{12}$$

Solutions of equations 11 and 12 are accordingly β_+ and β_-

$$e^{i\beta_\pm L} = \frac{1}{2(\tilde{t}_{xx} \pm \tilde{r}_{xy})}\left[1 + \tilde{t}_{xx}^2 - \tilde{r}_{xx}^2 \pm 2\tilde{r}_{xy}(\tilde{t}_{xx} - \tilde{r}_{xx}) + \sqrt{S_\pm}\right] \tag{13}$$

where $S_\pm = 1 - 2\tilde{t}_{xx}^2 - 2\tilde{r}_{xx}^2 + \tilde{t}_{xx}^4 + \tilde{r}_{xx}^4 - 2\tilde{t}_{xx}^2\tilde{r}_{xx}^2 \mp 4\tilde{r}_{xy}(\tilde{t}_{xx} + \tilde{r}_{xx} + \tilde{r}_{xx}\tilde{t}_{xx}^2 + \tilde{t}_{xx}\tilde{r}_{xx}^2 - \tilde{t}_{xx}^3 - \tilde{r}_{xx}^3) + 4\tilde{r}_{xy}^2(\tilde{t}_{xx}^2 + \tilde{r}_{xx}^2 - 2\tilde{t}_{xx}\tilde{r}_{xx} - 1)$.

Eigen waves of structure are line polarized. Polarizations of eigen waves are mutually orthogonal. Amplitudes of field of eigen wave that correspond to solution $\beta_\pm$ of dispersion equation have values

$$A^j_{+x} = \pm A^j_{+y} = c_\pm, \quad A^j_{-x} = \pm A^j_{-y} = \frac{c_\pm}{2(\tilde{r}_{xx} \pm \tilde{r}_{xy})}\left[1 + \tilde{r}_{xx}^2 - \tilde{t}_{xx}^2 \mp 2\tilde{r}_{xy}(\tilde{t}_{xx} - \tilde{r}_{xx}) \pm \sqrt{S_\pm}\right] \tag{14}$$

Value $c_\pm$ is arbitrary constant.

4. Numerical Results

There are line polarized eigen waves only in the more complicated structures consist of arrays with C, S or Ω-shaped strip particles also. Two eigen waves are orthogonal polarized and have different phase velocities and different stop band frequencies.

Dependence of propagation constants of eigen waves in the structure of S-shaped strip particles from distance between layers is shown in Fig. 2. Directions of polarization of eigen waves don't vary when distance between arrays is varied.

One eigen wave is polarized at the angle approximately equal to 56.7 degrees regard to Ox axis. The wave has strong dependence of propagation constant from distance between arrays at resonant frequency region. There are stop band zones of this wave. The width of zone is increased with increasing reflection of single array. If frequency lower than resonant frequency a phase velocity of this wave is more than light velocity and it is smaller than light velocity in opposite case.

Other eigen wave marked by sign $\perp$ in Fig. 2 is polarized at the angle approximately equal to -33.3 degrees in regard to Ox axis. This wave has propagation constant the same as one in free space.

Similar characteristics have eigen waves of C-shaped and Ω-shaped layered structures. Both eigen waves of these structures can have stop band frequency zones.

5. Conclusion

If sizes of array cell are little than wave length in free space only two eigen waves can propagate in periodic structures of complex arrays of plane particles having any shape. The eigen waves have linearly polarized mutually orthogonal fields.

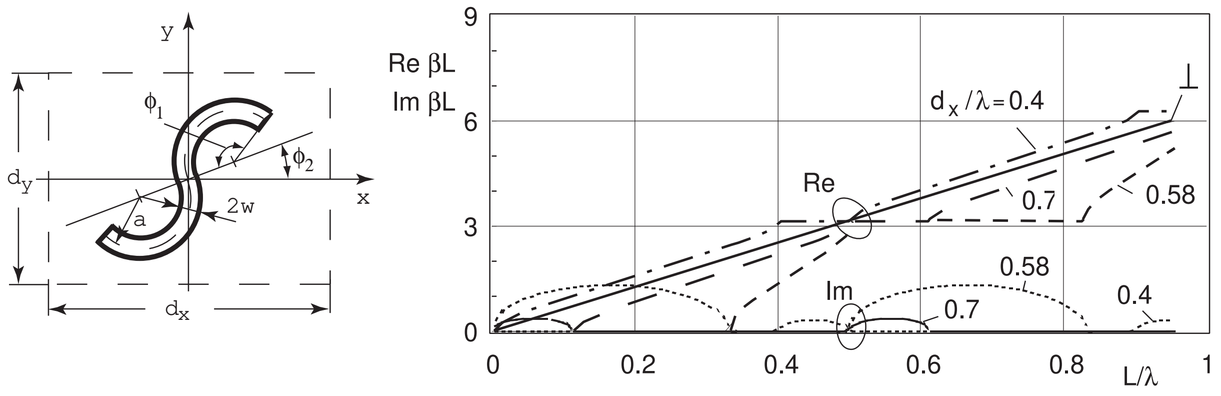

Figure 2: Propagation constant β versus distance between layers: array without substrate $h = 0$, $d_x = d_y = 10$ mm, $a = 3$ mm, $\phi_1 = \pi/2$, $\phi_2 = 0$, $w = 0.05$ mm, $d_x/\lambda_0 \approx 0.58$ is resonant value.

The field of the one eigen wave in the plano-chiral structure of S-shaped strips is polarized linearly along an average direct line that is similar to the segment of the straight line in a symbol *$*. The field of another eigen wave is polarized orthogonal to this direction. The polarization of eigen waves don't vary if a distance between layers is varied. The stop band zones is extended at increasing of the single array reflection. The phase velocity of eigen wave can be both more or less than the light velocity depending on the frequency that is lower or higher than the resonance frequency of a strip element of array.

Transformation of polarization doesn't occur at reflection and transmission of normal incident plane wave from single array or multi-layer array if incident wave has line polarization coincident with polarization of eigen wave of infinite structure.

Array of any plane particles has property to transform polarization of normal incident wave the same as an array of cross-shaped plane particles oriented along directions of polarizations of eigen waves on condition that reflection and transmission coefficients of both arrays are equal in selected frequency.

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