Analysis of Reflection of Electromagnetic Waves by Multi-Layered Arrays of Complex-Shaped Elements: Application to Electronically Controllable PBG

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Abstract

The results of the numerical analysis of reflecting and transmitting properties of multi-layered structures composed of doubly periodic arrays of metallic strips are presented in the paper. Strip elements have been selected, having a shape of letters C, S and Ω. The strips of the arrays are placed on a thin dielectric substrate. Strip elements can have bulk impedance loading, included in the break of strip. The interference of multi-layered array systems, due to multiple reflections of waves between layers, combined with the resonance properties of single layers lead the structure to acquire properties typical for photonic band gap crystals (PBG).

1. Introduction

Photonic band gap (PBG) crystals made with a few layers of periodic arrays of metal strip particles placed on thin dielectric substrate are useful for a wide range of applications in the microwave region such as novel antenna structures, frequency selective surfaces, filters with alternate frequency stop bands and pass bands. Frequent PBG structures are constructed with arrays of thin PEC disks [1] and thin metal rods [2]. A complex shape of strip particles of PBG structures gives various new resources. First of all, a complex shape particle can have a total length greater than the size of array's unit cell. This is important for making substrates of tiny microwave devices. Secondly, PBG consisting of complex shape particles can effectively transform the polarization of reflected wave in comparison with polarization of incident wave. Structures of plano-chiral elements such as strip having the shape of letter S have properties similar to true chiral structures [3]. Third of advantages of complex shaped particles is controllability of their electromagnetic properties by connecting active electronic loads such as PIN diodes to the complex particles.

The reflection properties of complex layered metal strip arrays placed in free space was studied recently [4]. Now our main goal is the study of reflection of more practical PBG layered array structures on dielectric substrates and arrays consisting of strip particles having bulk impedance loading, included in the break of strip.

2. Operators of Reflection and Transmission of Single Array Layer

Let us assume a plane electromagnetic wave incident on an plane double periodic array. The reflected and transmitted fields can be represented as superposition of partial waves of TE and TM polarizations. It is convenient to confront set of amplitudes of reflected partial waves and transmitted ones with a set of amplitudes of incident field by using operators $r_1^{\pm}$ and $t_1^{\pm}$ of reflection and transmission a single array placed on dielectric substrate. The indices plus and minus respectively are denoted the operators for the wave incident from the side of metal strip and opposite side of dielectric slab. Method of moments was used practically in the cases of wave scattering by arrays of thin narrow strips. The method of works [5], [6] was modified for simulation wave scattering by array on substrate.

3. Operators of Reflection and Transmission of a Finite Number of Arrays

A system of a finite number of arrays is shown in Fig. 1. The structure is assumed to be equidistant and to consist of identical arrays having identical orientations of the elements. Electromagnetic field in each gap between planar arrays may be represented in the form of a set of partial TE - and TM - waves propagating or exponentially decaying from one array plane to another. Amplitudes of the transverse components of the partial waves are denoted as following: q for the incident field, $r_n^+ q$ for the reflected field, $t_n^+ q$ for the transmitted field, and A, B for the fields in the gap between the next to the last array and the last array of the structure, see Fig. 1.

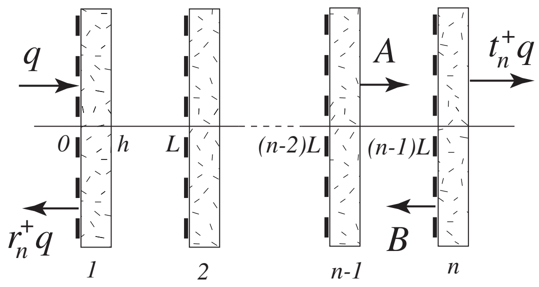

Figure 1: Layered structure

Let us assume operators $r_1^{\pm}$, $t_1^{\pm}$ for a single array to be known, as well as $r_{n-1}^{\pm}$, $t_{n-1}^{\pm}$ for the system of $(n-1)$ arrays, and show that the operators for the whole system can then be found recursively. The amplitudes of the partial waves satisfy equations

$$\left\{\begin{array}{lcl} A & = & t_{n-1}^{+} q + r_{n-1}^{-} e B \\ B & = & r_1^{+} e A \\ r_n^{+} q & = & r_{n-1}^{+} q + t_{n-1}^{-} e B \\ t_n^{+} q & = & t_1^{+} e A \end{array}\right. \tag{1}$$

where operator e is the plane-wave propagator operator from the plane of one array to the next array plane along the direction of propagation. The quite similar equations can be wrote in the case of electromagnetic wave incidence from right side of layered structure. After elimination of vectors A and B from (1) and the corresponding vectors of second set of equations one obtains recurrent expressions which allow to find operators $r_n^{\pm}$ and $t_n^{\pm}$ in the form

$$\begin{array}{lcl} r_n^{\pm} & = & r_{n-1}^{\pm} + t_{n-1}^{\mp} e r_1^{\pm} e (I - r_{n-1}^{\mp} e r_1^{\pm} e)^{-1} t_{n-1}^{\pm} \\ t_n^{\pm} & = & t_1^{\pm} e (I - r_{n-1}^{\mp} e r_1^{\pm} e)^{-1} t_{n-1}^{\pm} \end{array} \tag{2}$$

In this way we can find any scattering characteristics of the array system, i.e. amplitudes of partial waves reflected and transmitted by the layered system using operators $r_n^{\pm}$ and $t_n^{\pm}$.

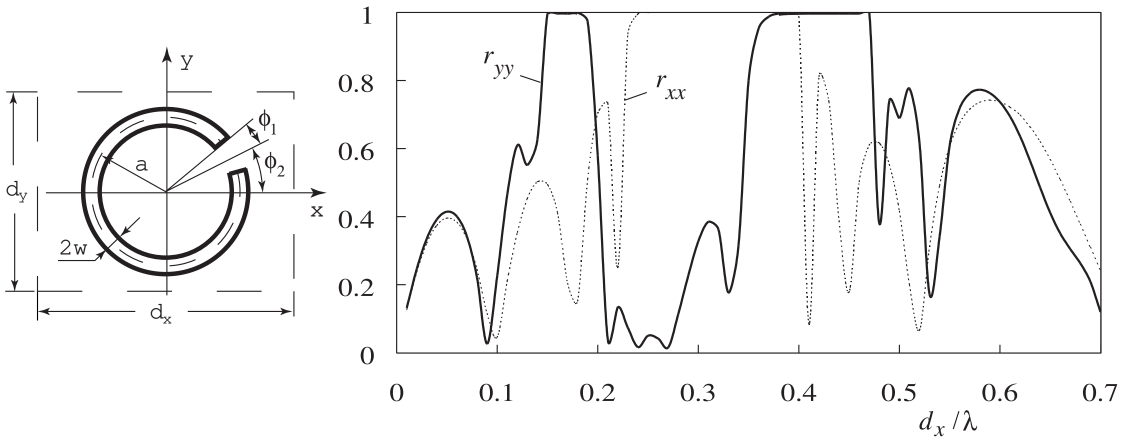

Figure 2: Absolute values of the reflection coefficients: C-shaped particles, 4 layers, $\varepsilon = 3$, $h = 0.25$ mm, $d_x = d_y = 3$ mm, $a = 1.25$ mm, $\phi_1 = 10°$, $\phi_2 = 0$, $w = 0.05$ mm, $L = 2.5$ mm

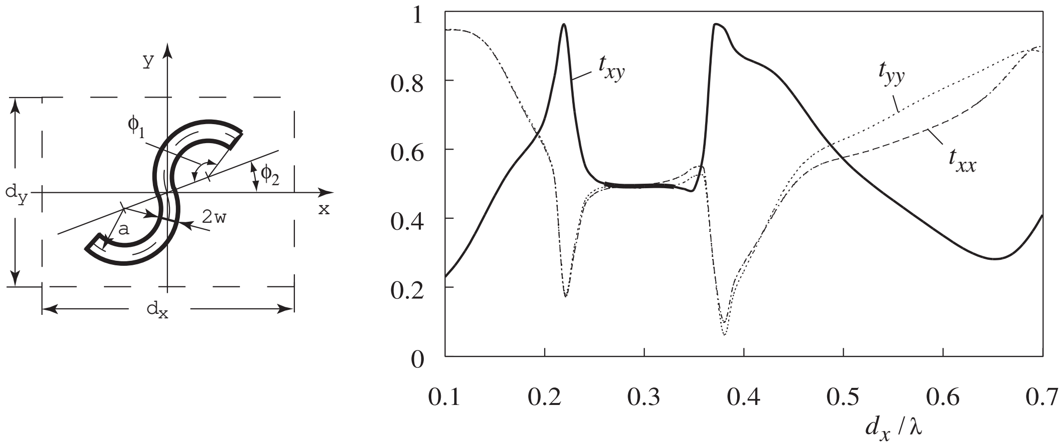

Figure 3: Absolute values of the transmission coefficients: S-shaped particles, 3 layers, $\varepsilon = 3$, $h = 0.25$ mm, $d_x = d_y = 3$ mm, $a = 1$ mm, $\phi_1 = 120°$, $\phi_2 = 0$, $w = 0.05$ mm, $L = 2.5$ mm

4. Numerical Results and Discussion

A system with two wide zones of full reflection can be made using a 4-layer structure with C-shaped particles on substrate. Its frequency characteristics are shown on Fig. 2. The first reflection zone is in the low frequency area. It is the first resonance (polarization along axis Oy) depending on the element length with the resonance between the first and the fourth layers. The second zone is the second length resonance with resonances between layers: layer 1 and layer 2 ($L_{12} \approx \lambda/2$), layer 1 and layer 3 ($L_{13} \approx \lambda$), layer 1 and layer 4 ($L_{14} \approx 3\lambda/2$), when $d_x/\lambda \approx 0.6$.

Similar behavior of frequency dependence of the reflection coefficient is observed in the case of a 4-layer structure with Ω-shaped elements.

Layered arrays of plano-chiral S-shaped particles have ability of effective transformation of polarization of incident wave near resonant frequency of strip particle. There is a wide frequency zone of equal levels of absolute values of transmission and reflection coefficients, see Fig. 3.

Frequency dependencies of reflection coefficients of layered arrays of loaded particles are shown on Fig. 4. Inductive loading leads to increase the electric length of particle. The resulting particle resonances and the resonances due to interference between layers give a complex system

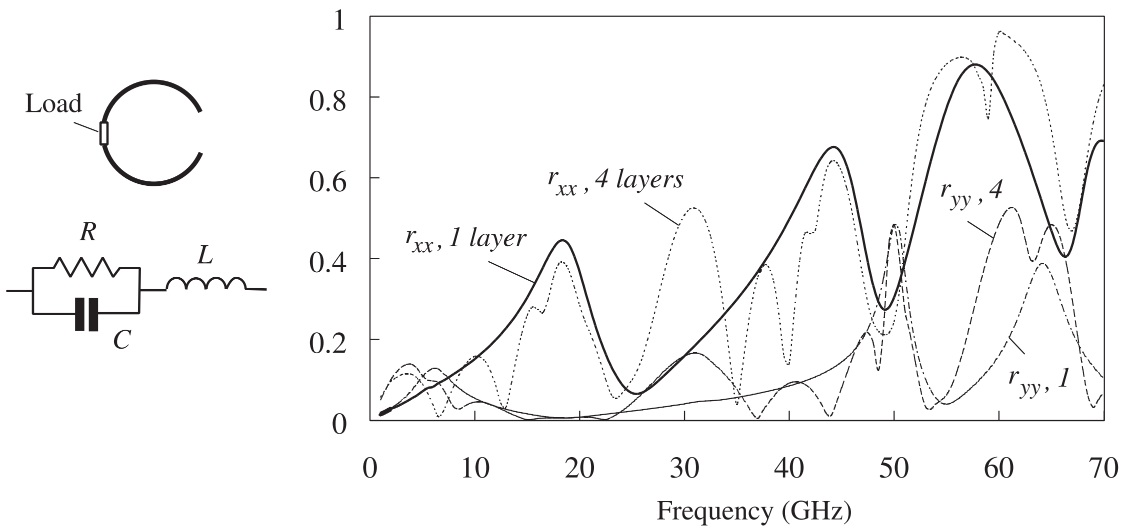

Figure 4: Absolute values of the reflection coefficients: C-shaped loaded particles, array without substrate, $d_x = d_y = 6$ mm, $a = 2.5$ mm, $\phi_1 = 10^\circ$, $\phi_2 = 0$, $w = 0.1$ mm, $L = 5$ mm, parameters of load are $R = 0.136\ \Omega$, $L = 1.2$ pH, $C = 0$ F

of reflection and transmission zones. There is possibility to control frequency characteristics of structure by using active electronic devices as load.

4. Conclusion

Using layered structures instead of single array of complex shape particles enable to give the opportunity to have more sharp and wide band frequency zones of full reflection and polarization transformation. Controlling the frequency characteristics of layered arrays can be achieved by associating electronic loads to the array particles.

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