Electromagnetic Wave Propagation in Circular Waveguide Containing Chiral Rod

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Abstract

Here we study the scattering properties of the section of circular waveguide containing lossy isotropic chiral rod. The effects of the chirality on the polarisation state (electromagnetic field rotation and axial ratio) of fundamental mode propagate in the guide are discussed and verified experimentally.

1. Introduction

The effects of the chirality on the polarization state of the waves propagating in the circular guides filled with chiral medium have been intensively studied in the last few years [1] $\div$ [6]. Such an effect allows the determination of the chiral material parameters and can be utilized in a number of novel applications. The theoretical and experimental investigation of the scattering properties of the chiral guide section allows the development of inversion method to determine the constitutive chiral material parameters [3, 4, 5]. In this paper we investigate analytically the problem of the circular waveguide containing a lossy chiral rod. The relevant research includes the complete description of the modal spectrum. Furthermore the scattering matrix of the chiral section is derived using the mode matching approach. The analysed junction consists of the chiral section and two transitions connecting this section with empty circular waveguides. In our experiments we use the chiral rod for which the constitutive parameters are closed to reported in [3]. The scattering characteristics were investigated and subsequently applied to calculate the polarisation state parameters of the fundamental waves transmitted through the chiral section. In the proof of principle experiment the performance of a chiral structure was measured and good agreement with theory was obtained.

2. Theory

The cross section of the chiral circular guide here analysed is shown in Figure 1*a*. The structure consists of the helix loaded chiral rod of length 20 mm and radius 5 mm, which is introduced into teflon sleeve entirely filling the cylindrical guide of the radius 10 mm. The chiral rods were fabricated in Sowerby Research Centre British Aerospace, England by dispersing stainless steel helices in an epoxy resin matrix. The helices have three turns of 1mm outside diameter, and a pith of 0.5 mm. The wire gauge is 0.15 mm. The rod contains 70 helices in total giving the metallic fraction of 0.6 %. In our experiments we use the values of the chiral material parameters as permeability, permittivity and chirality shown in Figure 2. It is closed to presented in [3] where they are extracted for similar chiral medium.

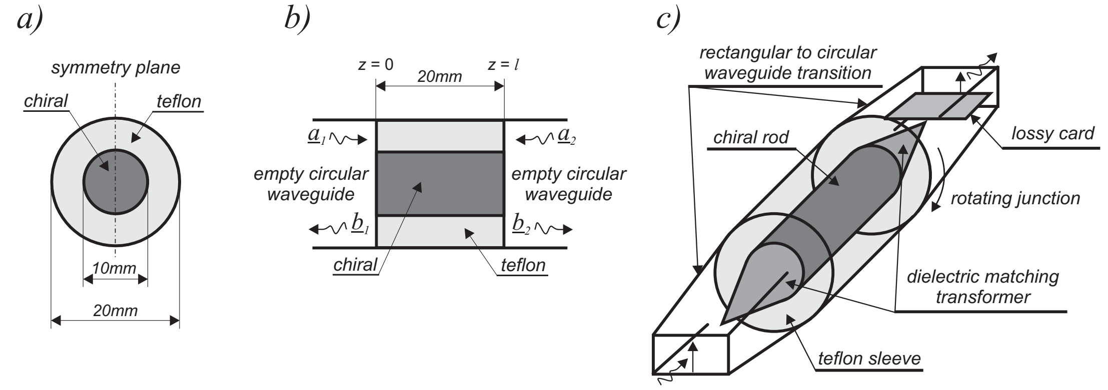

Figure 1: Cylindrical guide containing lossy chiral rod: *a)* cross–section; *b)* investigated structure; *c)* experimental setup

The transfer matrix procedure proposed in [6] for general solution of the circular waveguide containing chiroferrite rods has been applied to derive rigorous dispersion relation for the considered guide. The dispersion characteristics of the two fundamental left (LCP)and right (RCP) circularly polarised modes appearing in the guide are shown in the Figure 3. The differences observed between the modes propagation coefficients indicate that chiral material responds differentially to left and right handed circularly polarised waves. It means that the polarisation state of the wave propagated along the chiral guide is changes.

To determine the scattering matrix of a lossy chiral rod inside a circular waveguide the structure is subdivided as shown in Figure 1*b* into two transitions from the empty to the chiral

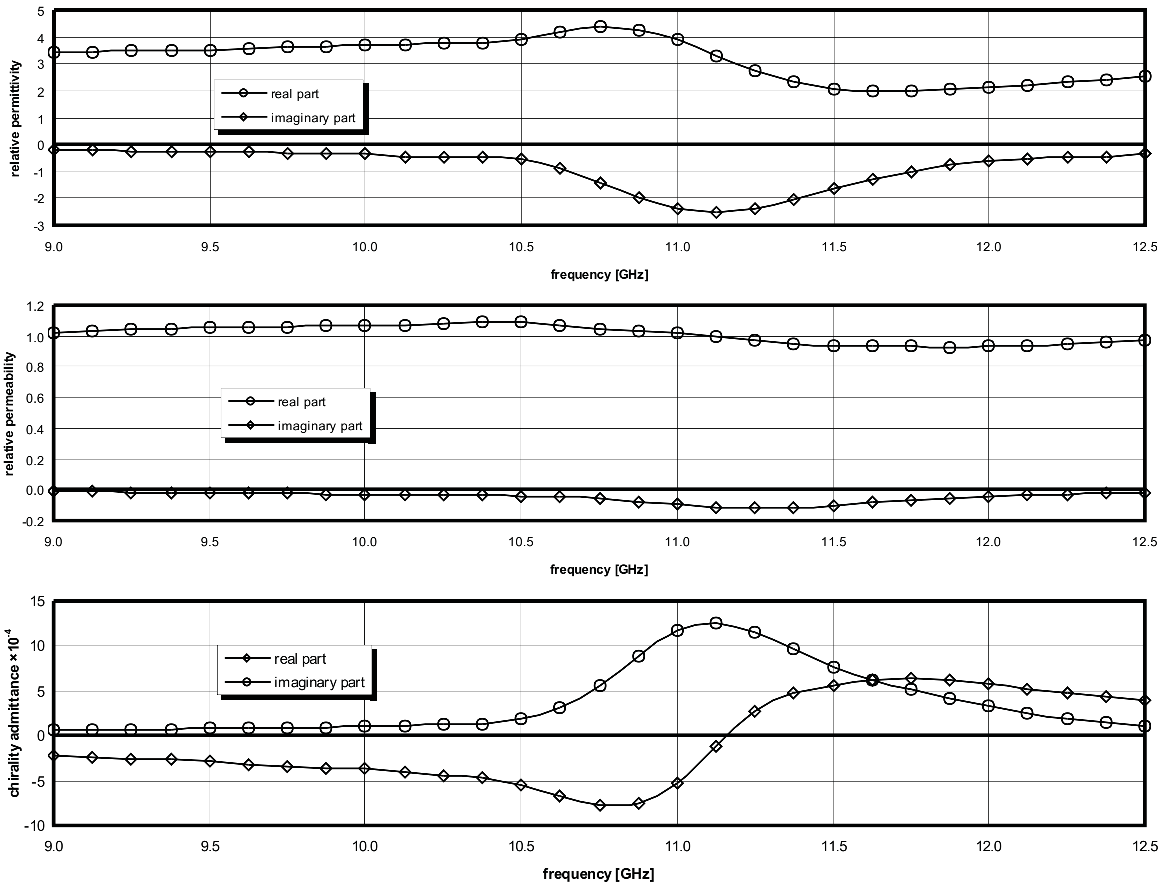

Figure 2: Constitutive parameters of the chiral material (data follows from [3])

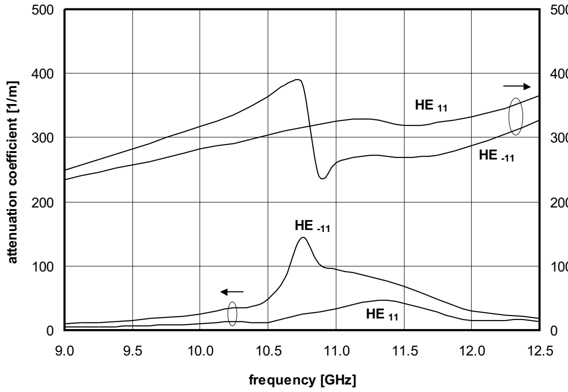

Figure 3: Dispersion characteristics of the fundamental modes in considered circular guide

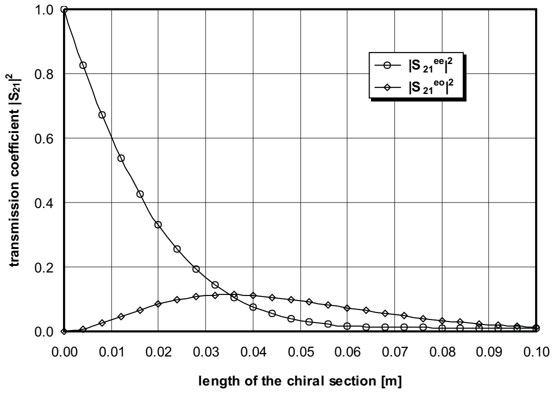

Figure 4: Transmission coefficients versus length of the chiral section

section at $z = 0$ and $z = l$ and the connecting chiral guide. In the current problem we assume that only two circularly polarised modes can propagate in the chiral section and the empty waveguides can support only two orthogonal TE_{11} modes being co–polar (even) and cross–polar (odd) with respect to the chosen symmetry plane of the guide cross–section. Imposing the boundary conditions for the tangential electric and magnetic fields at interfaces $z = 0$ and $z = l$ and orthogonalizing the fields with respect to the modes of the lossless empty waveguide, leads to the set of eight linear algebraic equations from which the scattering matrix is obtained as:

$$\begin{bmatrix} b_1^e \\ b_1^o \\ b_2^e \\ b_2^o \end{bmatrix} = \begin{bmatrix} S_{11}^{ee} & S_{11}^{oe} & S_{12}^{ee} & S_{12}^{oe} \\ S_{11}^{eo} & S_{11}^{oo} & S_{12}^{eo} & S_{12}^{oo} \\ S_{21}^{ee} & S_{21}^{oe} & S_{22}^{ee} & S_{22}^{oe} \\ S_{21}^{eo} & S_{21}^{oo} & S_{22}^{eo} & S_{22}^{oo} \end{bmatrix} \begin{bmatrix} a_1^e \\ a_1^o \\ a_2^e \\ a_2^o \end{bmatrix} \tag{1}$$

Here the superscripts and the subscripts refer to the empty waveguide co–polar (e) and cross–polar (o) modes and port respectively. One should note from Figure 4 that in dependence of the chiral section length the variation of the transmission coefficients S_{21}^{ee} and S_{21}^{eo} is observed. It means that the input of dominant co–polar TE_{11} mode at port (1) causes the output at the port (2) both fundamental co– and cross–polar modes. This coupling effect is possible because the rotation phenomenon occurs in the chiral waveguide section. Moreover the equal values of $S_{21}^{ee(oo)}$ and $S_{12}^{ee(oo)}$ indicate the reciprocal behaviour of the considered structure. The knowledge of the S_{21}^{ee} and S_{21}^{eo} elements allows to determine the polarization state parameters of the wave transmitted across the chiral section. From Stokes equations defined as [2, 7]:

$$\begin{aligned} S_0 &= S_{21}^{ee} S_{21}^{ee^*} + S_{21}^{eo} S_{21}^{eo^*} \;\; ; \quad S_2 = -2\,Re\Big(S_{21}^{ee} S_{21}^{eo^*}\Big) \\ S_1 &= S_{21}^{ee} S_{21}^{ee^*} - S_{21}^{eo} S_{21}^{eo^*} \;\; ; \quad S_3 = 2\,Im\Big(S_{21}^{ee} S_{21}^{eo^*}\Big) \end{aligned} \tag{2}$$

the rotation angle is:

$$\theta = \frac{1}{2}\,arctan\left(\frac{S_2}{S_1}\right) \tag{3}$$

and the axial ratio is written as:

$$AR = tan\left[\frac{1}{2}\,arcsin\left(\frac{S_3}{S_0}\right)\right] \tag{4}$$

3. Numerical and Experimental Results

Figure 1c shows the experimental setup used for measurement of polarization state parameters of the wave propagated along the considered chiral guide. The section of chiral circular guide is

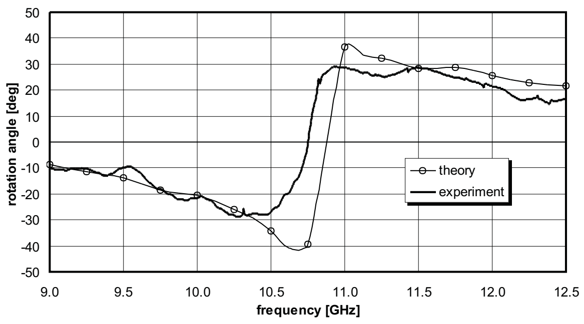

Figure 5: Characteristics of the rotation angle

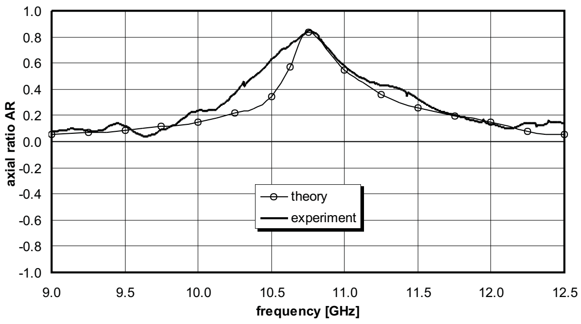

Figure 6: Characteristics of the axial ratio

placed between two circular to rectangular waveguide transitions with tapered dielectric transformers matching the chiral rod. For characterization of the polarization state parameters of the wave transmitted across the chiral sample the measurement of co– and cross–polar transmission coefficients are required. The co–polar transmission coefficient is derived when the input and output rectangular waveguides are parallel. The cross–polar coefficient is measured when output waveguide is rotated 90° relative to the input waveguide. The setup (Hewlett–Packard) was calibrated using the TRL calibration in the co–polarization configuration.

Figure 5 shows the theoretical and experimental frequency dependent characteristics of the angle of rotation for the considered chiral guide of the length 20 mm. The change of the sign of rotation angle above the resonance frequency is observed. The resonance occurs near the frequency 10.75 GHz. This frequency is in agreement to the half–wavelength resonance frequency 10.8 GHz of the current in the helix wire. As shown in Figure 6, where axial ratio characteristics are presented, near the resonance frequency the axial ratio approaches the value of 0.8. Here, the wave at the output of the chiral section is nearly circularly polarised.

4. Conclusion

The experimental results agree well with the theoretical prediction for both examined polarization state parameters. It means that the proposed solution sufficiently describes the scattering properties of the considered chiral waveguide.

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