A Hybrid FDFD-BIE Approach to Two-dimensional Scattering from an Inhomogeneous Biisotropic Cylinder

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Abstract

The scattering problem for an inhomogeneous two-dimensional biisotropic cylinder is solved in the frequency-domain by means of a hybrid method, in which finite difference equations in the interior region are combined with a mesh truncation in terms of a boundary integral equation. Numerical results for the bistatic echo widths are presented and compared with a reference solutions in the circular cylinder cases and it is found that the method yields more accurate results than what can be achieved with a local absorbing boundary condition.

1. Introduction

A scattering problem involving inhomogeneous media can be solved using a volume integral equation (VIE) that is discretized into a matrix equation by using the method of moments (MoM). However, a drawback with VIE is that the integral operator discretizes into a filled matrix, that becomes very large and exceedingly time consuming to invert if the geometry is several wavelengths in size. An alternative is to use a partial differential equation (PDE) method, in which the differential operator discretizes into a sparse matrix by using finite differences (FD) or by using the finite element method (FEM). However, when using FD or FEM in an open region a radiation condition must be supplemented by truncating the computational domain with an absorbing boundary condition (ABC), which can be either an approximate (local) ABC or an exact (global) ABC. One example of a local ABC is the perfectly matched layer (PML), which has been extended recently to include bianisotropic media [1]. A global ABC is usually formulated in terms of an integral operator on the boundary of the computational domain, which yields that the FD or FEM equations then must be solved in conjunction with a boundary integral equation (BIE). Examples of such combined methods, referred to as hybrid methods, are e.g. FEM-BIE [2] and FDTD-BIE [3]. In this paper, we consider a hybrid FD FrequencyDomain-BIE approach to a two-dimensional scattering problem for an inhomogeneous cylinder of a biisotropic material. Interior to the region containing the biisotropic cylinder, we use finite differences and on the boundary of the region we use a BIE, which reduces to a contour integral equation.

2. The Scattering Problem

Consider an in free space located biisotropic scatterer (see figure 1) described by the following constitutive relations in the frequency-domain:

$$\boldsymbol{D}(\boldsymbol{r}) = \varepsilon_0 \varepsilon(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}) + \sqrt{\varepsilon_0 \mu_0} \xi(\boldsymbol{r}) \boldsymbol{H}(\boldsymbol{r}), \qquad \boldsymbol{B}(\boldsymbol{r}) = \mu_0 \mu(\boldsymbol{r}) \boldsymbol{H}(\boldsymbol{r}) + \sqrt{\varepsilon_0 \mu_0} \zeta(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}), \quad (1)$$

where the parameters ε, μ, ξ and ζ are arbitrary functions of the position variable $\boldsymbol{r}$.

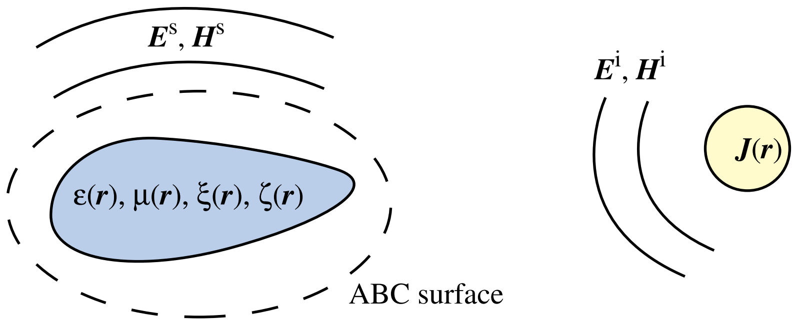

Figure 1: The scattering configuration.

The scattering problem is to determine the scattered fields $\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}$ when the scatterer is excited by known incident fields $\boldsymbol{E}^{\mathrm{i}}, \boldsymbol{H}^{\mathrm{i}}$, created by an extraneous time-harmonic $(\exp(\mathrm{j}\omega t))$ current density $\boldsymbol{J}$; see figure 1. Substituting the constitutive relations (1) into the Maxwell's equations, we eventually arrive at the following equations for the scattered fields:

$$\nabla \times \boldsymbol{E}^{\mathrm{s}} + \mathrm{j}k_0 \left(\mu\eta_0 \boldsymbol{H}^{\mathrm{s}} + \zeta \boldsymbol{E}^{\mathrm{s}}\right) = -\mathrm{j}k_0 \left(\left(\mu - 1\right)\eta_0 \boldsymbol{H}^{\mathrm{i}} + \zeta \boldsymbol{E}^{\mathrm{i}}\right), \tag{2}$$

$$\nabla \times \eta_0 \boldsymbol{H}^{\mathrm{s}} - \mathrm{j}k_0 \left(\varepsilon \boldsymbol{E}^{\mathrm{s}} + \xi\eta_0 \boldsymbol{H}^{\mathrm{s}}\right) = \mathrm{j}k_0 \left(\left(\varepsilon - 1\right)\boldsymbol{E}^{\mathrm{i}} + \xi\eta_0 \boldsymbol{H}^{\mathrm{i}}\right). \tag{3}$$

3. The FDFD Equations in the Two-dimensional Case

The 2D FDFD equations are derived in a similar way as in [4]. The region containing the scatterer is divided into square cells (see figure 2), in which the constitutive parameters are approximated as homogeneous and equal to their true values at the mid points. In the outermost layer of cells, the parameters are $\varepsilon = \mu = 1, \xi = \zeta = 0$. The FDFD approximation results in a linear system of equations for the everywhere continuous z-components of the fields [5]:

$$\mathbf{M}'_{\mathrm{EE}} \bar{E}^{\mathrm{s}}_{z,\mathrm{inside}} + \mathbf{B} \bar{E}^{\mathrm{s}}_{z,\mathrm{outside}} + \mathbf{M}_{\mathrm{EH}} \bar{H}^{\mathrm{s}}_{z,\mathrm{inside}} = \bar{J}^{\mathrm{E}}_{\mathrm{inside}}, \tag{4}$$

$$\mathbf{M}'_{\mathrm{HH}} \bar{H}^{\mathrm{s}}_{z,\mathrm{inside}} + \mathbf{B} \bar{H}^{\mathrm{s}}_{z,\mathrm{outside}} + \mathbf{M}_{\mathrm{HE}} \bar{E}^{\mathrm{s}}_{z,\mathrm{inside}} = \bar{J}^{\mathrm{H}}_{\mathrm{inside}}, \tag{5}$$

where $\bar{E}^{\mathrm{s}}_{z,\mathrm{inside}}, \bar{H}^{\mathrm{s}}_{z,\mathrm{inside}}$ are evaluated at the interior FDFD points, indicated with filled circles in figure 2, and $\bar{E}^{\mathrm{s}}_{z,\mathrm{outside}}, \bar{H}^{\mathrm{s}}_{z,\mathrm{outside}}$ are evaluated at the extruding FDFD points, indicated with blank circles in figure 2. $\mathbf{M}'_{\mathrm{EE}}, \mathbf{M}_{\mathrm{EH}}, \mathbf{M}_{\mathrm{HE}}$ and $\mathbf{M}'_{\mathrm{HH}}$ are sparsely filled square matrices, describing the FDFD operators at the interior points and the matrix $\mathbf{B}$ describes the FDFD operators on the extruding points. $\bar{J}^{\mathrm{E}}_{\mathrm{inside}}$ and $\bar{J}^{\mathrm{H}}_{\mathrm{inside}}$ are excitation vectors, obtained as linear combinations of the components of the incident fields at the interior points.

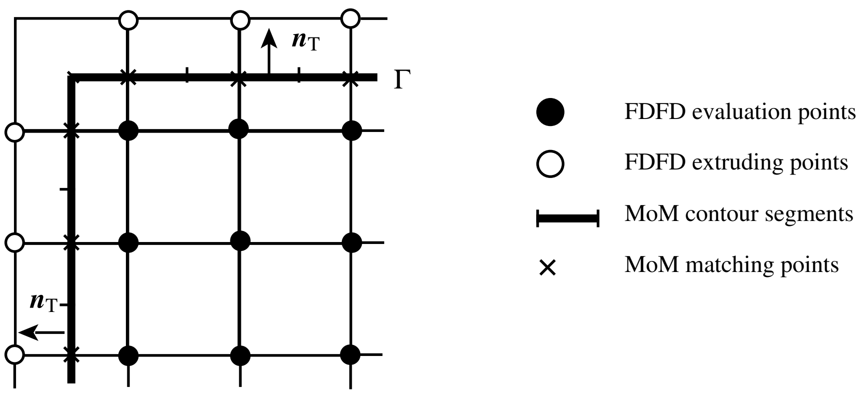

Figure 2: One corner region of the computational domain. Γ is the contour at which the global absorbing boundary condition is calculated.

4. Absorbing Boundary Condition in Terms of a BIE

With all their sources inside the contour Γ, depicted with a thick solid line in figure 2, the z-component of the scattered fields satisfy the integral equation

$$\frac{1}{2}F_z^{\mathrm{s}}(\boldsymbol{r}) + \oint_\Gamma \left[G(\boldsymbol{r},\boldsymbol{r}')\frac{\partial F_z^{\mathrm{s}}}{\partial n'}(\boldsymbol{r}') - F_z^{\mathrm{s}}(\boldsymbol{r}')\frac{\partial G}{\partial n'}(\boldsymbol{r},\boldsymbol{r}')\right]\mathrm{d}\Gamma' = 0, \quad \boldsymbol{r} \text{ on } \Gamma, \tag{6}$$

where F_z^{s} denotes E_z^{s} or H_z^{s} and $G(\boldsymbol{r},\boldsymbol{r}') = -\frac{j}{4}\mathrm{H}_0^{(2)}(k_0|\boldsymbol{r}-\boldsymbol{r}'|)$ is the 2D free space Green's function. After a simple MoM approximation using pulse functions and point-matching at the points indicated with crosses in figure 2, we obtain the following relation (the numerical ABC) between the scattered field and its normal derivative:

$$\mathbf{P}\bar{F}_z^{\mathrm{s}} = \mathbf{Q}\partial_n\bar{F}_z^{\mathrm{s}}, \tag{7}$$

where $\bar{F}_z^{\mathrm{s}}$ and $\partial_n\bar{F}_z^{\mathrm{s}}$ are vectors containing the field and its normal derivative, respectively, and where $\mathbf{P}$ and $\mathbf{Q}$ are matrices. To connect the ABC to the FDFD scheme, the elements in $\bar{F}_z^{\mathrm{s}}$ and $\partial_n\bar{F}_z^{\mathrm{s}}$ are approximated from the two nearest FDFD points on each side of Γ by linear interpolations and central differences, respectively, i.e.

$$F_z^{\mathrm{s}} \approx \frac{F_{z,\text{outside}}^{\mathrm{s}} + F_{z,\text{inside}}^{\mathrm{s}}}{2}, \qquad \partial_n F_z^{\mathrm{s}} \approx \frac{F_{z,\text{outside}}^{\mathrm{s}} - F_{z,\text{inside}}^{\mathrm{s}}}{h}, \tag{8}$$

where h (the mesh parameter) is the side-length of a cell. With (8) in (7), we obtain a matrix relation between the field values at the different side of Γ:

$$\mathbf{D}\bar{E}_{z,\text{outside}}^{\mathrm{s}} = \mathbf{C}\bar{E}_{z,\text{inside}}^{\mathrm{s}}, \qquad \mathbf{D}\bar{H}_{z,\text{outside}}^{\mathrm{s}} = \mathbf{C}\bar{H}_{z,\text{inside}}^{\mathrm{s}}, \tag{9}$$

where the square matrix $\mathbf{D}$ appears to be well-conditioned and hence invertible. Using (9) in (4) and (5), it thus follows that the fields at the interior points are obtained from the following matrix equation:

$$\begin{bmatrix} \mathbf{M}_{\mathrm{EE}} & \mathbf{M}_{\mathrm{EH}} \\ \mathbf{M}_{\mathrm{HE}} & \mathbf{M}_{\mathrm{HH}} \end{bmatrix} \cdot \begin{bmatrix} \bar{E}_{z,\text{inside}}^{\mathrm{s}} \\ \bar{H}_{z,\text{inside}}^{\mathrm{s}} \end{bmatrix} = \begin{bmatrix} \bar{J}_{\text{inside}}^{\mathrm{E}} \\ \bar{J}_{\text{inside}}^{\mathrm{H}} \end{bmatrix}, \tag{10}$$

where

$$\mathbf{M}_{\mathrm{EE}} = \mathbf{M}'_{\mathrm{EE}} + \mathbf{B}\mathbf{D}^{-1}\mathbf{C}, \qquad \mathbf{M}_{\mathrm{HH}} = \mathbf{M}'_{\mathrm{HH}} + \mathbf{B}\mathbf{D}^{-1}\mathbf{C}. \tag{11}$$

Note that since neither $\mathbf{B}$ nor $\mathbf{C}$ nor $\mathbf{D}$ depend on the properties of the scatterer, the operation $\mathbf{B}\mathbf{D}^{-1}\mathbf{C}$ in (11) has to be carried out only once. Thus, for a given mesh and frequency the ABC can be precalculated and stored for further usage when considering different properties of the scatterer, as long as the scatterer is confined within the boundary Γ.

5. Numerical Example

In this one example, consider a circular chiral cylinder with the radius $= \lambda$, where λ is the wavelength in free space. The constitutive parameters are chosen to be $\varepsilon = 2.6 - 0.2\mathrm{j}, \mu = 1.5 - 0.1\mathrm{j}, \xi = -\zeta = -\mathrm{j}\kappa = -\mathrm{j}(0.6 - 0.1\mathrm{j})$. For the computational region we consider two different mesh sizes, 41×41 FDFD points with spacing $h = 0.05\lambda$ and 81×81 FDFD points with spacing $h = 0.025\lambda$. The numerical results are compared with a reference solution obtained by means of an expansion in cylindrical eigenwaves. The cross-polarized bistatic echo width $\sigma_{\text{cross}}^{\mathrm{E}} = \sigma_{\text{cross}}^{\mathrm{H}}$ for waves impinging in the direction $\phi = 0$ is depicted in figure 3, with the solid lines for the reference solution, the dotted lines for $h = 0.05\lambda$ and the dashed lines for $h = 0.025\lambda$. The numerical results indicate convergence and are in good agreements with the reference solution.

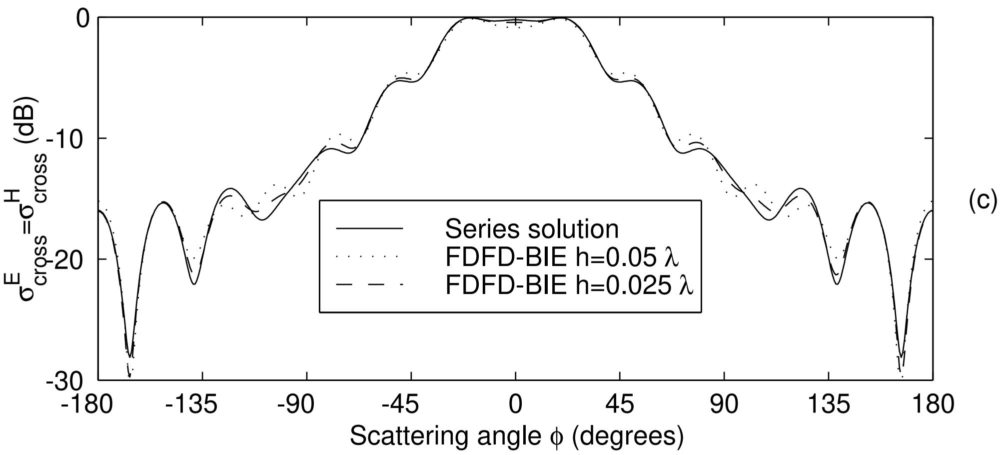

Figure 3: The cross-polarized bistatic echo width for a chiral circular cylinder with the radius $= \lambda$ and the parameters $\varepsilon = 2.6 - 0.2\mathrm{j}, \mu = 1.5 - 0.1\mathrm{j}$ and $\kappa = 0.6 - 0.1\mathrm{j}$. The cylinder is illuminated in the direction $\phi = 0°$.

6. Conclusion

The scattering problem for inhomogeneous two-dimensional biisotropic cylinders has been considered. Finite difference equations for the interior region as well as a contour integral equation realizing a global absorbing boundary condition have been derived and implemented numerically.

Computationally, the present FDFD-BIE method is much faster than a moment method using a filled matrix of the same size. Hence, the present method can be used for solving 2D scattering problems involving scatterers of intermediate sizes (i.e. a couple of wavelengths) in both directions whereas MoM under, the same computation time, only can be used for considerably smaller or thinner structures.

Since our global ABC is unlikely to generate artifacts from the boundary in the solution, the usefulness of the hybrid method as a fast solver in an optimization approach for solving the corresponding inverse problem is of interest for forthcoming research.

References

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