Interaction of Bianisotropic Particles and Energy Conservation in Regular Arrays

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Abstract

Interaction of bianisotropic particles in plane regular arrays is under investigation in this paper. We give a recipe on how to use the particle polarizabilities and the interaction constants obtained from approximate models so that the energy balance is satisfied and the physics of the phenomena is kept. Numerical examples are given for arrays of omega particles.

1. Introduction

To solve a diffraction problem for an array of scattering particles one should, at first, know the properties of an isolated inclusion given by its polarizability and, second, how the inclusions interact in the array. The polarizabilities as well as the interaction fields often cannot be calculated exactly. The aim of this paper is to give a method which will allow us to use the polarizabilities and the interaction coefficients obtained by approximate models so that the reflection and transmission coefficients will satisfy the energy conservation principle.

2. Energy Conservation in Bianisotropic Arrays

Consider a plane regular array of scattering particles. We will assume that the particles may be represented as combinations of electric and magnetic dipoles. Thus, every particle is characterized by its dyadic polarizability factors $\overline{\overline{\alpha}}$:

$$\begin{array}{rcl} \mathbf{p} & = & \overline{\overline{\alpha}}_{ee} \cdot \mathbf{E}_{\rm loc} + \overline{\overline{\alpha}}_{em} \cdot \mathbf{H}_{\rm loc} \\ \mathbf{m} & = & \overline{\overline{\alpha}}_{me} \cdot \mathbf{E}_{\rm loc} + \overline{\overline{\alpha}}_{mm} \cdot \mathbf{H}_{\rm loc} \end{array} \tag{1}$$

We assume that the array is excited by a normally incident plane wave with the fields $\mathbf{E}_{\rm ext}$ and $\mathbf{H}_{\rm ext}$. Every particle is excited by the local fields

$$\mathbf{E}_{\rm loc} = \mathbf{E}_{\rm ext} + \overline{\overline{\beta}}_e \cdot \mathbf{p}, \qquad \mathbf{H}_{\rm loc} = \mathbf{H}_{\rm ext} + \overline{\overline{\beta}}_m \cdot \mathbf{m} \tag{2}$$

Here $\overline{\overline{\beta}}_e$ and $\overline{\overline{\beta}}_m$ are the interaction dyadics. These dyadics take into account interaction of the particles in the array. Under our assumptions the array of electric dipoles does not produce any magnetic interaction field and the array of magnetic dipoles does not produce any electric interaction field. Due to this there are no cross terms in (2). As it was shown in our recent work [1] the interaction dyadics for the considered problem can be represented as

$$\overline{\overline{\beta}}_e = \mathrm{Re}(\overline{\overline{\beta}}_e) + j\frac{\eta\epsilon_0\mu_0\omega^3}{6\pi}\overline{\overline{I}} - j\frac{\eta\omega}{2S_0}\overline{\overline{I}}_t, \qquad \overline{\overline{\beta}}_m = \mathrm{Re}(\overline{\overline{\beta}}_m) + j\frac{\epsilon_0\mu_0\omega^3}{6\pi\eta}\overline{\overline{I}} - j\frac{\omega}{2\,\eta S_0}\overline{\overline{I}}_t \tag{3}$$

Here S_0 is unit cell area. As one can see, the last terms in the above relations correspond to the plane wave field contribution. We can express the local fields in terms of the induced dipole moments:

$$\begin{aligned} \mathbf{E}_{\rm loc} &= \left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1}\cdot(\mathbf{p} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\mathbf{m}) \\ \mathbf{H}_{\rm loc} &= \left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1}\cdot(\mathbf{m} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\mathbf{p}) \end{aligned} \tag{4}$$

The external fields, as follows from (2), can be written as

$$\begin{aligned} \mathbf{E}_{\rm ext} &= \left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1}\cdot(\mathbf{p} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\mathbf{m}) - \overline{\overline{\beta}}_e\cdot\mathbf{p} \\ \mathbf{H}_{\rm ext} &= \left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1}\cdot(\mathbf{m} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\mathbf{p}) - \overline{\overline{\beta}}_m\cdot\mathbf{m} \end{aligned} \tag{5}$$

The total averaged fields (plane wave fields) in the array plane read

$$\begin{aligned} \mathbf{E}_{\rm tot} &= \mathbf{E}_{\rm ext} - j\tfrac{\eta}{2}\tfrac{\omega}{S_0}\overline{\overline{I}}_t\cdot\mathbf{p} = \overline{\overline{Z}}_{ee}\cdot\mathbf{J} + \overline{\overline{Z}}_{em}\cdot\mathbf{J}_m \\ \mathbf{H}_{\rm tot} &= \mathbf{H}_{\rm ext} - j\tfrac{1}{2\eta}\tfrac{\omega}{S_0}\overline{\overline{I}}_t\cdot\mathbf{m} = \overline{\overline{Z}}_{me}\cdot\mathbf{J} + \overline{\overline{Z}}_{mm}\cdot\mathbf{J}_m \end{aligned} \tag{6}$$

Here, the currents $\mathbf{J}$ and $\mathbf{J}_m$ are not the average surface electric and magnetic currents. These vectors can be arbitrarily directed and they represent the normalized electric and magnetic dipole moments. The dyadic coefficients in (6) can be easily identified from the above formulas:

$$\overline{\overline{Z}}_{ee} = -j\frac{S_0}{\omega}\left[\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1} - \overline{\overline{\beta}}_e - j\frac{\eta}{2}\frac{\omega}{S_0}\overline{\overline{I}}_t\right] \tag{7}$$

$$\overline{\overline{Z}}_{mm} = -j\frac{S_0}{\omega}\left[\left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1} - \overline{\overline{\beta}}_m - j\frac{1}{2\eta}\frac{\omega}{S_0}\overline{\overline{I}}_t\right] \tag{8}$$

$$\overline{\overline{Z}}_{em} = j\frac{S_0}{\omega}\left[\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1}\cdot\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\right] \tag{9}$$

$$\overline{\overline{Z}}_{me} = j\frac{S_0}{\omega}\left[\left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1}\cdot\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\right] \tag{10}$$

Although different terms have different dimensions, we use the same notation Z for all of them. Indeed, only $\overline{\overline{Z}}_{ee}$ has the meaning of impedance.

Let us now suppose that the particles have no dissipation losses. Then, the energy conservation condition

$$\mathrm{Re}\{\mathbf{E}_{\rm tot}\cdot\mathbf{J}^* + \mathbf{J}_m\cdot\mathbf{H}_{\rm tot}^*\} = 0 \tag{11}$$

can be written in the dyadic form as

$$\begin{pmatrix}\mathbf{J}\\ \mathbf{J}_m\end{pmatrix}\cdot\begin{pmatrix}\overline{\overline{Z}}_{ee}^T + \overline{\overline{Z}}_{ee}^* & \overline{\overline{Z}}_{em}^* + \overline{\overline{Z}}_{me}^T \\ \overline{\overline{Z}}_{em}^T + \overline{\overline{Z}}_{me}^* & \overline{\overline{Z}}_{mm}^T + \overline{\overline{Z}}_{mm}^*\end{pmatrix}\cdot\begin{pmatrix}\mathbf{J}^*\\ \mathbf{J}_m^*\end{pmatrix} = 0 \tag{12}$$

Since this should be valid identically for all currents, we find that in lossless arrays

$$\overline{\overline{Z}}_{ee} + \overline{\overline{Z}}_{ee}^\dagger = 0, \qquad \overline{\overline{Z}}_{mm} + \overline{\overline{Z}}_{mm}^\dagger = 0 \tag{13}$$

$$\overline{\overline{Z}}_{em} + \overline{\overline{Z}}_{me}^{\dagger} = 0 \tag{14}$$

where † denotes the Hermite conjugate. Consider reciprocal particles. Then $\overline{\overline{Z}}_{ee}$ and $\overline{\overline{Z}}_{mm}$ are symmetric dyadics. Thus, (13) means that these dyadics are purely imaginary (and the bracketed expressions are purely real). In other words,

$$\mathrm{Im}\left\{\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1}\right\} = \frac{\eta\epsilon_0\mu_0\omega^3}{6\pi}\overline{\overline{I}} \tag{15}$$

$$\mathrm{Im}\left\{\left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1}\right\} = \frac{\epsilon_0\mu_0\omega^3}{6\pi\eta}\overline{\overline{I}} \tag{16}$$

and similarly from (14):

$$\mathrm{Re}\left\{\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\cdot\overline{\overline{\alpha}}_{me}\right)^{-1}\cdot\overline{\overline{\alpha}}_{em}\cdot\overline{\overline{\alpha}}_{mm}^{-1}\right\} = 0 \tag{17}$$

$$\mathrm{Re}\left\{\left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\cdot\overline{\overline{\alpha}}_{em}\right)^{-1}\cdot\overline{\overline{\alpha}}_{me}\cdot\overline{\overline{\alpha}}_{ee}^{-1}\right\} = 0 \tag{18}$$

The last two relations are equivalent since in reciprocal media $\overline{\overline{Z}}_{me} = -\overline{\overline{Z}}_{em}^{T}$. For a special case of omega particles with

$$\overline{\overline{\alpha}}_{ee} = a_{ee}^{xx}\mathbf{x}_0\mathbf{x}_0 + a_{ee}^{yy}\mathbf{y}_0\mathbf{y}_0, \quad \overline{\overline{\alpha}}_{mm} = a_{mm}\mathbf{z}_0\mathbf{z}_0, \quad \overline{\overline{\alpha}}_{me} = a_{me}\mathbf{z}_0\mathbf{y}_0, \quad \overline{\overline{\alpha}}_{em} = -a_{me}\mathbf{y}_0\mathbf{z}_0 \tag{19}$$

relations (15) and (16) give

$$\begin{gathered}\mathrm{Im}\left\{\frac{1}{a_{ee}^{xx}}\right\} = \frac{\eta\epsilon_0\mu_0\omega^3}{6\pi}, \qquad \mathrm{Im}\left\{\frac{a_{mm}}{a_{ee}^{yy}a_{mm} + a_{me}^2}\right\} = \frac{\eta\epsilon_0\mu_0\omega^3}{6\pi} \\ \mathrm{Im}\left\{\frac{a_{ee}^{yy}}{a_{ee}^{yy}a_{mm} + a_{me}^2}\right\} = \frac{\epsilon_0\mu_0\omega^3}{6\pi\eta}\end{gathered} \tag{20}$$

Conditions (17) and (18) lead to the same result, which reads

$$\mathrm{Re}\left\{\frac{a_{me}}{a_{ee}^{yy}a_{mm} + a_{me}^2}\right\} = 0 \tag{21}$$

3. Reflection and Transmission Coefficients

Let us now make the final step. Our main goal is to find the reflection and transmission coefficients. It is easy to see that they can be found in terms of the introduced parameters (7)–(10). The parameters are expressed via the particle polarizabilities. In the usual practice we have the polarizabilities dyadics found from the antenna model or numerically. This gives approximate results and the values of alphas are not quite correct. We should somehow "correct" the polarizabilities to find the reflection and transmission coefficients which satisfy the energy conservation law.

From the other hand, the correction should not lead to significant difference in the array reflective properties. If we simply skip the scattering terms from the polarizabilities and the interaction dyadics, i.e. we take α_{ee} and α_{mm} as purely real, α_{em} and α_{me} as purely imaginary, and leave only plane wave contribution in the imaginary parts of betas, we easily satisfy the energy conservation conditions (15)–(18). But the frequency behaviour of the array reflection will change dramatically. It follows from (7)–(8): equating the scattering terms in the alphas to zero we change the values of dyadics $\overline{\overline{Z}}$. It suggests us to apply additional conditions to avoid that. These conditions will keep the non-scattering terms of (7)–(8) unchanged when the scattering terms in alphas and betas are skipped. Doing so we obtain a system of equations on the new "corrected" polarizabilities $\overline{\overline{\alpha}}'_{ee}$, $\overline{\overline{\alpha}}'_{mm}$, $\overline{\overline{\alpha}}'_{em}$, and $\overline{\overline{\alpha}}'_{me}$. In more detail, the system is

$$\mathrm{Im}(\overline{\overline{\alpha}}'_{ee}) = \mathrm{Im}(\overline{\overline{\alpha}}'_{mm}) = 0, \qquad \mathrm{Re}(\overline{\overline{\alpha}}'_{em}) = 0 \tag{22}$$

and

$$\begin{aligned}
\left(\overline{\overline{\alpha}}'_{ee} - \overline{\overline{\alpha}}'_{em} \cdot {\overline{\overline{\alpha}}'_{mm}}^{-1} \cdot \overline{\overline{\alpha}}'_{me}\right)^{-1} &= \mathrm{Re}\left\{\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em} \cdot \overline{\overline{\alpha}}_{mm}^{-1} \cdot \overline{\overline{\alpha}}_{me}\right)^{-1}\right\} \\
\left(\overline{\overline{\alpha}}'_{mm} - \overline{\overline{\alpha}}'_{me} \cdot {\overline{\overline{\alpha}}'_{ee}}^{-1} \cdot \overline{\overline{\alpha}}'_{em}\right)^{-1} &= \mathrm{Re}\left\{\left(\overline{\overline{\alpha}}_{mm} - \overline{\overline{\alpha}}_{me} \cdot \overline{\overline{\alpha}}_{ee}^{-1} \cdot \overline{\overline{\alpha}}_{em}\right)^{-1}\right\} \\
j\left(\overline{\overline{\alpha}}'_{ee} - \overline{\overline{\alpha}}'_{em} \cdot {\overline{\overline{\alpha}}'_{mm}}^{-1} \cdot \overline{\overline{\alpha}}'_{me}\right)^{-1} \cdot \overline{\overline{\alpha}}'_{em} \cdot {\overline{\overline{\alpha}}'_{mm}}^{-1} &= \\
-\mathrm{Im}\left\{\left(\overline{\overline{\alpha}}_{ee} - \overline{\overline{\alpha}}_{em} \cdot \overline{\overline{\alpha}}_{mm}^{-1} \cdot \overline{\overline{\alpha}}_{me}\right)^{-1} \cdot \overline{\overline{\alpha}}_{em} \cdot \overline{\overline{\alpha}}_{mm}^{-1}\right\}&
\end{aligned} \tag{23}$$

The "corrected" polarizabilities together with the "corrected" betas (only plane wave contribution is included) will give the reflection and transmission satisfying the energy conservation law.

4. Numerical Results

We have numerically investigated the case of a double array of omega particles under the plane wave excitation. The polarizabilities were obtained from the antenna model described in [2]. The reflection coefficient values via the frequency together with the energy balance plot are given on Figure 1 and Figure 2. The solid lines correspond to the corrected alphas and betas, the dashed lines correspond to the original ones.

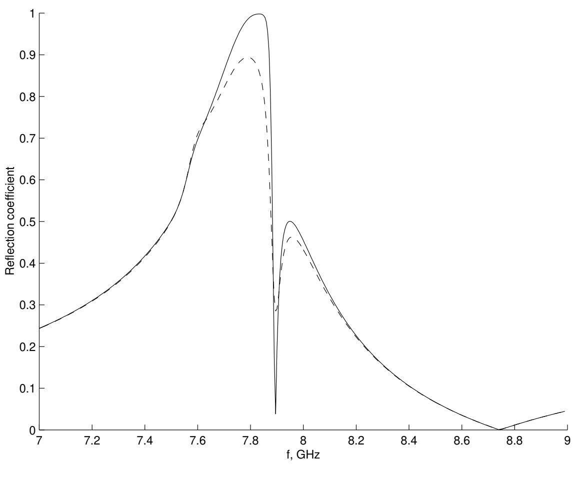

Figure 1: Reflection coefficient as function of frequency.

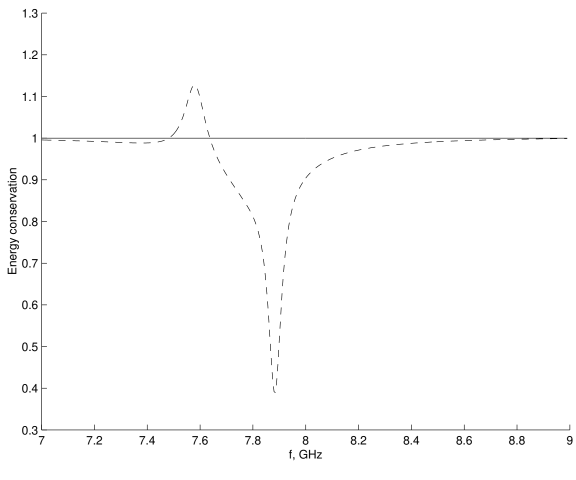

Figure 2: Energy conservation as function of frequency.

References

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