Frequency Selective Structures on a Bianisotropic Slab

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Abstract

In this paper a method for the analysis of a frequency selective surface (FSS) supported by a bianisotropic substrate is presented. The frequency selective structure is a thin metallic pattern—the actual FSS—on a plane supporting substrate. Integral representations of the fields in combination with the method of moments carried out in the spatial Fourier domain are shown to be a fruitful way of analyzing the problem with a complex substrate.

1. General Equations

The geometry of interest in this paper is depicted in Figure 1. The sources of the problem are assumed to be confined to a region located to the left of the bianisotropic slab, which extends from $z = z_1$ to $z = z_{N-1}$. The depth parameter z is defined by the normal of the interfaces as shown in the figure. The scatterer is a periodic pattern of metal—frequency selective surface (FSS)—located at $z = z_0$ on the left hand side of the slab. The space outside the slab is assumed to be homogeneous, lossless and isotropic with relative permittivity ϵ, permeability μ, and relative impedance $\eta = \sqrt{\mu/\epsilon}$.

The integral representation of the solution to the Maxwell equations in an isotropic region is used to characterize the electric field in the region outside the slab and the scatterer. The stratified geometry also suggest that an expansion of the Green's dyadic in plane vector waves is pertinent [1]. A systematic use of these two concept gives the following representations of the scattered electric field [3] ($\eta_0 = \sqrt{\mu_0/\epsilon_0}$ and $k = \sqrt{\epsilon\mu}\omega/c_0$):

$$\boldsymbol{E}^s(\boldsymbol{r}) = \begin{cases} -\dfrac{1}{4\pi^2}\displaystyle\iint_{-\infty}^{\infty}\left(\frac{k\eta_0\eta}{2k_z}\mathbf{P}^+(\boldsymbol{k}_t)\cdot\boldsymbol{J}(\boldsymbol{k}_t)e^{-ik_zz_0}\right)e^{i\boldsymbol{k}_t\cdot\boldsymbol{\rho}+ik_zz}\,dk_x\,dk_y \\ \qquad -\dfrac{1}{4\pi^2}\displaystyle\iint_{-\infty}^{\infty}\left(\boldsymbol{\gamma}^-\cdot\mathbf{r}(\boldsymbol{k}_t)\cdot\boldsymbol{F}^+(\boldsymbol{k}_t,z_1)e^{ik_zz_1}\right)e^{i\boldsymbol{k}_t\cdot\boldsymbol{\rho}-ik_zz}\,dk_x\,dk_y, \quad z_0 < z < z_1 \\ -\dfrac{1}{4\pi^2}\displaystyle\iint_{-\infty}^{\infty}\Big(\frac{k\eta_0\eta}{2k_z}\mathbf{P}^-(\boldsymbol{k}_t)\cdot\boldsymbol{J}(\boldsymbol{k}_t)e^{ik_zz_0} \\ \qquad +\boldsymbol{\gamma}^-(\boldsymbol{k}_t)\cdot\mathbf{r}(\boldsymbol{k}_t)\cdot\boldsymbol{F}^+(\boldsymbol{k}_t,z_1)e^{ik_zz_1}\Big)e^{i\boldsymbol{k}_t\cdot\boldsymbol{\rho}-ik_zz}\,dk_x\,dk_y, \quad z < z_0 \end{cases}$$

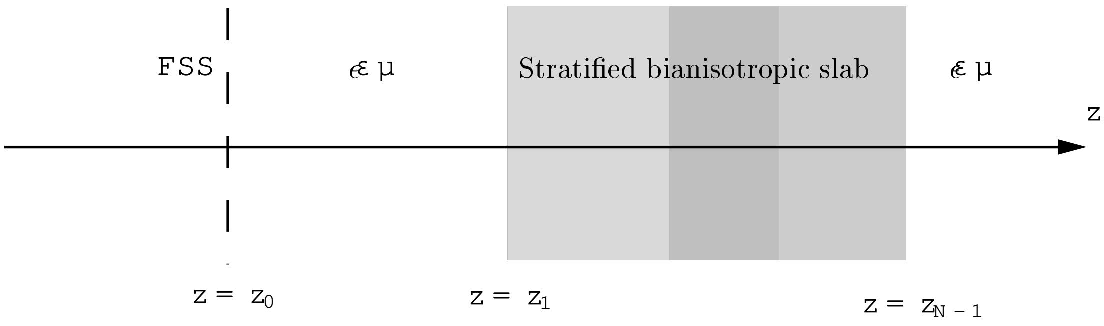

Figure 1: The geometry of the problem.

where the Fourier variable in the x-y-plane is denoted $\boldsymbol{k}_t$ and the normal (longitudinal) wave number, k_z, is defined by $k_z = \left(k^2 - k_t^2\right)^{1/2}$, $(\operatorname{Im} k_z \geq 0)$. $\boldsymbol{J}(\boldsymbol{k}_t)$ denotes the Fourier transformed surface currents of the plane $z = z_0$, and the projection dyadic $\mathbf{P}^{\pm}(\boldsymbol{k}_t)$ is defined by [3]

$$\mathbf{P}^{\pm}(\boldsymbol{k}_t) = \frac{k_z^2}{k^2}\hat{\boldsymbol{e}}_{||}\hat{\boldsymbol{e}}_{||} + \hat{\boldsymbol{e}}_{\perp}\hat{\boldsymbol{e}}_{\perp} \mp \frac{k_t k_z}{k^2}\left(\hat{\boldsymbol{z}}\hat{\boldsymbol{e}}_{||} + \hat{\boldsymbol{e}}_{||}\hat{\boldsymbol{z}}\right) + \frac{k_t^2}{k^2}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$$

and the reflection dyadic $\mathbf{r}(\boldsymbol{k}_t)$ of the slab and

$$\boldsymbol{\gamma}^{\pm}(\boldsymbol{k}_t) = \pm\left(\mathbf{I}_2 \mp \frac{k_t}{k_z}\hat{\boldsymbol{z}}\hat{\boldsymbol{e}}_{||}\right)$$

where $\mathbf{I}_2$ is the identity dyadic in the x-y-plane. The two (real) unit vectors in the x-y-plane

$$\hat{\boldsymbol{e}}_{||}(\boldsymbol{k}_t) = \boldsymbol{k}_t/k_t, \qquad \hat{\boldsymbol{e}}_{\perp}(\boldsymbol{k}_t) = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{e}}_{||}(\boldsymbol{k}_t)$$

and the split field $\boldsymbol{F}^{+}(\boldsymbol{k}_t, z)$ at the interface is [6]

$$\boldsymbol{F}^{+}(\boldsymbol{k}_t, z) = \frac{1}{2}\boldsymbol{E}_{xy}(\boldsymbol{k}_t, z) - \frac{\eta\eta_0}{2}\left(\hat{\boldsymbol{e}}_{||}\hat{\boldsymbol{e}}_{||}\frac{k}{k_z} + \ \mathit{unite}_{\perp}\hat{\boldsymbol{e}}_{\perp}\right) \cdot \hat{\boldsymbol{z}} \times \boldsymbol{H}_{xy}(\boldsymbol{k}_t, z)$$

2. Integral Equation for the Surface Current

We employ the Floquet's theorem [2] to the surface current $\boldsymbol{J}(\boldsymbol{r})$ on the FSS and the Fourier transform of this current is [5]

$$\boldsymbol{J}(\boldsymbol{k}_t) = \frac{4\pi^2}{A_E}\sum_{m,n=-\infty}^{\infty}\boldsymbol{J}_E(\boldsymbol{k}_{mn})\delta^2(\boldsymbol{k}_t - \boldsymbol{k}_{mn}), \quad \boldsymbol{k}_t \in \mathbb{R}^2$$

where A_E is the area of the unit cell (sides a and b with the angle Ω between the axis) and $\boldsymbol{k}_{mn} = \hat{\boldsymbol{x}}\alpha_m + \hat{\boldsymbol{y}}\beta_{mn}$ with

$$\alpha_m = \frac{2\pi m}{a} + k_x^i, \qquad \beta_{mn} = \frac{2\pi n}{b\sin\Omega} - \frac{2\pi m}{a}\cot\Omega + k_y^i$$

where k_x^i and k_y^i are the x- and the y-components of the wave vector of the incident field, respectively, and where $\boldsymbol{J}_E(\boldsymbol{k}_{mn})$ is the Fourier transform of $\boldsymbol{J}(\boldsymbol{r})$ over the unit cell E evaluated at $\boldsymbol{k}_{mn}$.

The boundary conditions on the FSS imply that

$$\left(\mathbf{I}_2 + \mathbf{r}(\boldsymbol{k}_{00})e^{2ik_{z\,00}h}\right) \cdot \boldsymbol{E}_{xy}^i(\boldsymbol{r})\big|_{z=z_0} = \sum_{m,n=-\infty}^{\infty}\left(\mathbf{I}_2 + \mathbf{r}(\boldsymbol{k}_{mn})e^{2ik_{z\,mn}h}\right) \cdot \boldsymbol{x}_{mn}e^{i\boldsymbol{k}_{mn}\cdot\boldsymbol{\rho}}$$

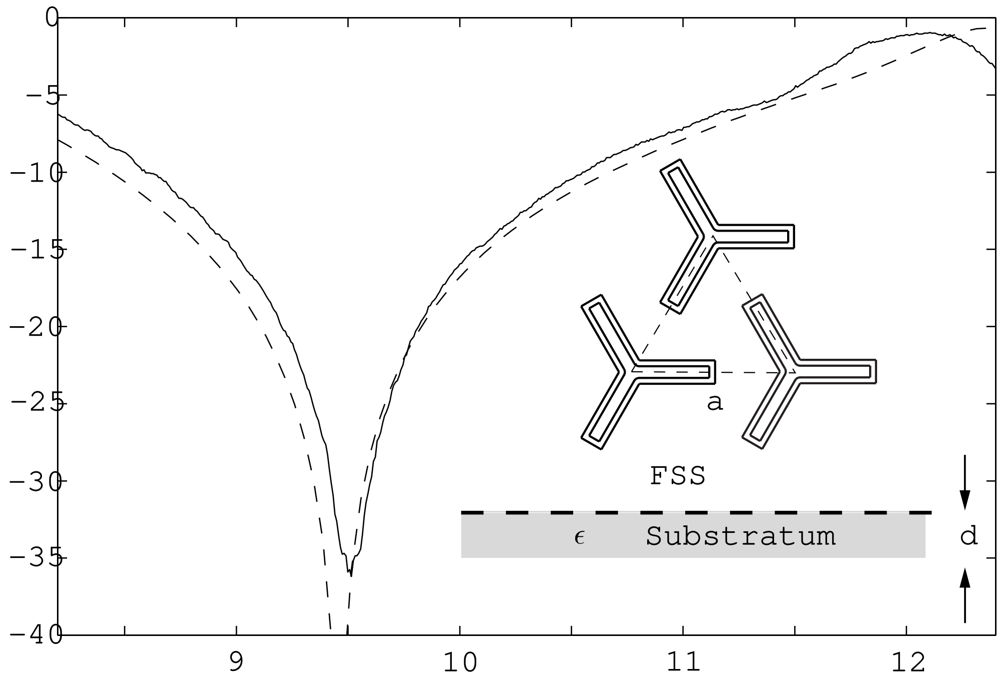

Figure 2: Power transmission (in dB scale) of the co-polarization for a hexagonal pattern of loaded tripoles on an isotropic slab as a function of frequency (GHz). The angle of incidence is $\theta = 60°$ and $\phi = 0°$, and the polarization is TE. The tripoles are 9 mm long with 3 mm long ends. The width of the metallic strips is 0.5 mm. The elements are arranged in an equilateral lattice with side 16.5 mm. The polarization of the incident field perpendicular with one of the sides in the hexagonal pattern. The thickness of the isotropic substrate is $d = 0.12$ mm and the permittivity is $\epsilon = 4.3(1 + i0.021)$. The dashed line shows the computed values and the solid line shows the measurements.

where $h = z_1 - z_0 > 0$ and where $k_{zmn} = \left(k^2 - |\boldsymbol{k}_{mn}|^2\right)^{1/2}$, $(\mathrm{Im}\, k_{zmn} \geq 0)$ and where we have introduced the vector field

$$\boldsymbol{x}_{mn} = \frac{k\eta_0\eta}{2A_E k_{zmn}} \left(\frac{k_z^2}{k^2}\hat{\boldsymbol{e}}_{\parallel}\hat{\boldsymbol{e}}_{\parallel} + \hat{\boldsymbol{e}}_{\perp}\hat{\boldsymbol{e}}_{\perp}\right) \cdot \boldsymbol{J}_E(\boldsymbol{k}_{mn})$$

to simplify the notation. This relation is the basic equation used for the determination of the unknown quantity $\boldsymbol{x}_{mn}$, which is solved by a method of moments technique in the spatial Fourier domain [3]. Once this quantity is determined, all other fields can be obtained.

3. Results

We illustrate the effect of an isotropic, homogeneous dielectric substrate on the transmission properties of the FSS in Figure 2. The effect of a bianisotropic substrate is illustrated in Figure 3. The constitutive relations used here are [6]

$$\boldsymbol{D} = \epsilon_0 \left\{\boldsymbol{\epsilon} \cdot \boldsymbol{E} + \eta_0 \boldsymbol{\xi} \cdot \boldsymbol{H}\right\}, \qquad \boldsymbol{B} = \frac{1}{c_0} \left\{\boldsymbol{\zeta} \cdot \boldsymbol{E} + \eta_0 \boldsymbol{\mu} \cdot \boldsymbol{H}\right\}$$

The material parameters of the slab is [4]

$$\boldsymbol{\epsilon} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad \boldsymbol{\mu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{\xi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\Omega \\ 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{\zeta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -i\Omega & 0 \end{pmatrix} \tag{1}$$

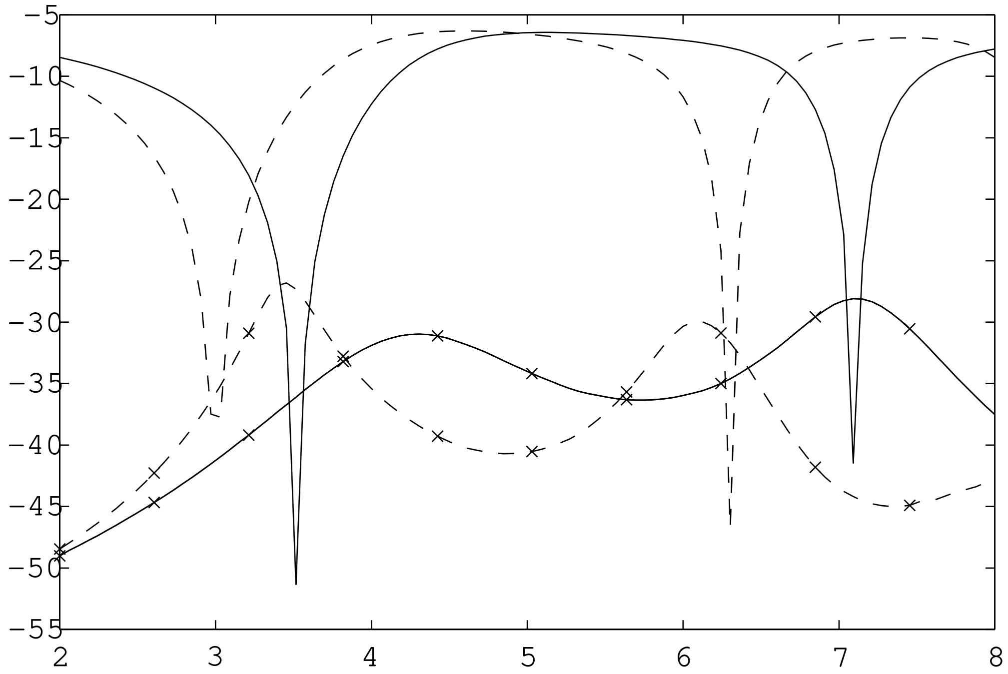

Figure 3: The same element and unit cell geometry as in Figure 2 but the substrate is bianisotropic. The material parameters is given in (1) and the thickness of the substrate is $d = 6$ mm. The curves that correspond to the co-polarization are given by lines without crosses and the cross-polarization curves are given by lines with crosses. The solid lines show the cases where $\epsilon_{yy} = 3$ and $\Omega = 0$ (*i.e.*, an isotropic substrate), and the dashed lines show the cases where $\epsilon_{yy} = 10$ and $\Omega = 0.9$. The angle of incidence is $\theta = 30^\circ$ and $\phi = 0^\circ$, and the polarization is TM.

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