Effects of Anisotropy in Light Scattering by Anisotropic Layer around a Spherical Particle in Uniaxial Medium

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Abstract

We have used a Mie–type theory to study the light scattering from an annular anisotropic layer around a spherical colloidal particle. We have derived an exact solution of the scattering problem in the case when the distribution of the optical axes around the particles posses some special transformation properties under rotation and, outside of the layer, the ambient medium is isotropic. We have then calculated the dependence of the the scattering cross-section on particle size, anisotropy parameter, and layer thickness for different optical axis distributions. We find that the scattering cross-section is strongly affected by the type of anisotropy. The presence of disclinations enhances scattering efficiency. We determine the region of validity of Rayleigh–Gans approximation by comparing approximate values of the scattering cross-section with the results computed from the exact solution. As an additional effect specific to anisotropic scatterer, it is found that for structures with broken central symmetry there is the phase shift proportional to the logarithm of layer thickness that enters the scattering amplitudes.

In order to study the case of anisotropic ambient medium, approximate theory has been developed. The phase shift is found to affect the scattering amplitudes even if the central symmetry is unbroken.

1. Introduction

There are a large number of physical contexts in which it is useful to understand light scattering by impurities [1]. A particular example of recent interest concerns liquid crystal devices. There are now a number of systems in which liquid crystal droplets are suspended in a polymer matrix – the so-called PDLC systems – or the inverse system, involving colloids now with a nematic liquid crystal solvent. The latter systems are commonly known as filled nematics [2, 3].

In such systems it is required to calculate the scattering of light by composite anisotropic particles embedded in an isotropic matrix. In this paper we discuss some model cases of light scattering by such particles, which may be supposed to represent local liquid crystalline director structures, using both the Mie and the Rayleigh-Gans (R-G) approaches.

The large majority of exact solutions of the single-scattering problem have been derived for isotropic scatterers [4]. However, there are a few cases for which Mie theory has been extended to the case of anisotropic scatterers[5, 6, 7, 8].

In order that the problem have an analytic solution, we find it necessary to restrict consideration to cases in which the optical axis distributions within the anisotropic layer around the core possess some special symmetry properties under rotation. The analysis is then based on a systematic expansion of the electromagnetic field over vector spherical harmonics [9, 10]. The specific form of the expansions is known as the T-matrix ansatz. This has been widely used in the related problem of light scattering by nonspherical particles [11].

In this study we investigate the dependence of light scattering from the layer on the internal structure of the optical tensor, and implicitly on the liquid crystal director texture. We also compare our exact results with those obtained by the simpler but less accurate R-G method. For brevity, in what follows we shall leave aside the details on this comparison and do not discuss extensions of the theory.

2. Scatterers

We consider scattering by a spherical particle of radius R_1 embedded in a uniform isotropic dielectric medium with dielectric constant $\epsilon_{ij} = \epsilon\delta_{ij}$ and magnetic permeability $\mu_{ij} = \mu\delta_{ij}$. The scattering particle consists of an inner isotropic core of radius R_2, surrounded by an anisotropic annular layer of thickness $d = R_1 - R_2$.

Within the inner core of the scatterer the dielectric tensor $\boldsymbol{\epsilon}$, and the magnetic permittivity $\boldsymbol{\mu}$ take the values $\boldsymbol{\epsilon}_{ij} = \epsilon_2\delta_{ij}$, $\boldsymbol{\mu}_{ij} = \mu_2\delta_{ij}$. The dielectric tensor within the annular layer is locally uniaxial. The optical axis distribution is defined by the vector field $\hat{\mathbf{n}}$. (Hats will denote unit vectors.) Then within the annular layer $\boldsymbol{\epsilon}_{ij}(\mathbf{r}) = \epsilon_1\delta_{ij} + \Delta\epsilon_1(\hat{\mathbf{n}}(\mathbf{r})\otimes\hat{\mathbf{n}}(\mathbf{r}))_{ij}$ and $\boldsymbol{\mu}_{ij} = \mu_1\delta_{ij}$. The unit vector $\hat{\mathbf{n}}$ corresponds to a liquid crystal director for material within the annular region.

We shall suppose that the director field can be written in the following form

$$\hat{\mathbf{n}} = n_r\hat{\mathbf{r}} + n_\varphi\hat{\boldsymbol{\varphi}} + n_\vartheta\hat{\boldsymbol{\vartheta}}, \tag{1}$$

where $\hat{\mathbf{r}}$ is the unit radial vector and $\hat{\boldsymbol{\varphi}}$, $\hat{\boldsymbol{\vartheta}}$ are the vectors tangential to the unit sphere. The components $n_r, n_\varphi, n_\vartheta$ are constants.

3. Generalized Mie Theory

The electromagnetic field can be expanded using the vector spherical harmonic basis, $\mathbf{Y}_{j+\delta\, j\, m}(\phi,\theta) \equiv \mathbf{Y}_{j+\delta\, j\, m}(\hat{\mathbf{r}})$ $(\delta = 0, \pm 1)$ [9] as follows:

$$\mathbf{E} = \sum_{jm}\mathbf{E}_{jm} = \sum_{jm}\left[p^{(0)}_{jm}(r)\mathbf{Y}^{(0)}_{jm}(\hat{\mathbf{r}}) + p^{(e)}_{jm}(r)\mathbf{Y}^{(e)}_{jm}(\hat{\mathbf{r}}) + p^{(m)}_{jm}(r)\mathbf{Y}^{(m)}_{jm}(\hat{\mathbf{r}})\right], \tag{2a}$$

$$\mathbf{H} = \sum_{jm}\mathbf{H}_{jm} = \sum_{jm}\left[q^{(0)}_{jm}(r)\mathbf{Y}^{(0)}_{jm}(\hat{\mathbf{r}}) + q^{(e)}_{jm}(r)\mathbf{Y}^{(e)}_{jm}(\hat{\mathbf{r}}) + q^{(m)}_{jm}(r)\mathbf{Y}^{(m)}_{jm}(\hat{\mathbf{r}})\right], \tag{2b}$$

where $\mathbf{Y}^{(m)}_{jm} = \mathbf{Y}_{j\,j\,m}$ and $\mathbf{Y}^{(e)}_{jm} = [j/(2j+1)]^{1/2}\mathbf{Y}_{j+1\,j\,m} + [(j+1)/(2j+1)]^{1/2}\mathbf{Y}_{j-1\,j\,m}$ are electrical and magnetic harmonics respectively, and $\mathbf{Y}^{(0)}_{jm} = [j/(2j+1)]^{1/2}\mathbf{Y}_{j-1\,j\,m} - [(j+1)/(2j+1)]^{1/2}\mathbf{Y}_{j+1\,j\,m}$ are longitudinal harmonics.

Outside the scatterer $p^{(\alpha)}_{jm}(r)$ and $q^{(\alpha)}_{jm}(r)$ can be expressed in terms of linear combinations of spherical Bessel functions and spherical Hankel functions. For the incident and scattered waves in the far field region we have:

$$\mathbf{E}_{inc} = \sum_{jm} \left(\alpha_{jm}^{(inc)} j_j(\rho) \mathbf{Y}_{jm}^{(m)} + \tilde{\alpha}_{jm}^{(inc)} \, D j_j(\rho) \mathbf{Y}_{jm}^{(e)} \right) \,, \tag{3}$$

$$\mathbf{E}_{sca} = \sum_{jm} \left(\beta_{jm}^{(sca)} h_j^{(1)}(\rho) \mathbf{Y}_{jm}^{(m)} + \tilde{\beta}_{jm}^{(sca)} \, D h_j^{(1)}(\rho) \mathbf{Y}_{jm}^{(e)} \right) \,, \tag{4}$$

where $\rho = kr$ and $Df(x) \equiv x^{-1} \dfrac{\mathrm{d}}{\mathrm{d}x}(xf(x))$. Since the scattering problem is linear, the coefficients $\{\beta_{jm}^{(sca)}, \tilde{\beta}_{jm}^{(sca)}\}$ are linearly related to $\{\alpha_{jm}^{(inc)}, \tilde{\alpha}_{jm}^{(inc)}\}$ through the elements of the T–matrix.

In order to derive the T–matrix, we need to find the general expressions for the electromagnetic field inside the anisotropic layer and the isotropic core. The ingoing and outgoing waves then can be related by using continuity of the tangential components of the electric and magnetic fields at $r = R_2$ and $r = R_1$ as boundary conditions.

Substituting the expansions (2) into the Maxwell equations gives a system of equations for the components $p_{jm}^{(\alpha)}$ and $q_{jm}^{(\alpha)}$. For the distributions (1) we have found that the system can be reduced to a system of coupled Bessel equations for the magnetic components: $\{p_{jm}^{(m)}, q_{jm}^{(m)}\}$. For the structures $\hat{\mathbf{n}} = \hat{\boldsymbol{\vartheta}}$ and $\hat{\mathbf{n}} = \cos\gamma \, \hat{\mathbf{r}} + \sin\gamma \, \hat{\boldsymbol{\varphi}}$ solutions of this system can be obtained in the closed form. The result for transverse components is

$$p_{jm}^{(m)} = \alpha_{jm} j_j(\rho_e) + \beta_{jm} h_j^{(1)}(\rho_e) \,, \quad q_{jm}^{(e)} = n_e/\mu_1 \, D p_{jm}^{(m)}(\rho_e) \,, \tag{5a}$$

$$q_{jm}^{(m)} \equiv \tilde{\rho}_e^{\,i\delta_j} \tilde{q}_{jm}^{(m)} = \tilde{\rho}_e^{\,i\delta_j} \left[\tilde{\alpha}_{jm} j_{\tilde{j}}(\tilde{\rho}_e) + \tilde{\beta}_{jm} h_{\tilde{j}}^{(1)}(\tilde{\rho}_e) \right] \,, \quad p_{jm}^{(e)} = -\mu_1/\tilde{n}_e \, \tilde{\rho}_e^{\,i\delta_j} \, D \tilde{q}_{jm}^{(m)}(\tilde{\rho}_e) \,, \tag{5b}$$

$$\tilde{j}(\tilde{j}+1) = \frac{1 + u_1(1 - n_\vartheta^2)}{(1 + u_1 n_r^2)^2} j(j+1), \quad \delta_j = \tilde{u}_1 \tilde{m}_e^2 \sqrt{j(j+1)} \, n_\varphi n_r, \tag{5c}$$

$$u_1 \equiv \Delta\epsilon_1/\epsilon_1, \quad m_e^2 = \frac{1 + u_1}{1 + u_1(1 - n_\vartheta^2)}, \quad \tilde{m}_e^2 = \frac{1 + u_1(1 - n_\vartheta^2)}{1 + u_1 n_r^2}, \tag{5d}$$

where α_{jm} and β_{jm} are integration constants; $\rho_e = m_e \rho_1 = m_e k_1 r$ and $\tilde{\rho}_e = \tilde{m}_e \rho_1$. For brevity, the corresponding rather cumbersome expressions for the T–matrix have been omitted. It is seen, that anisotropy affects the analytical expressions in the following manner: (a) it renormalizes the order of corresponding Bessel functions; (b) it changes the arguments of the functions by replacing k_1 with k_e; (c) it leads to the appearance of the geometric factor $\propto \rho^{i\delta_j}$ for the tilted configuration.

4. Scattering Efficiency

In this section we discuss briefly the results concerning the scattering efficiency, Q, that is the ratio of the total scattering cross section C_{sca} and area of the composite particle, $S = \pi R_1^2$.

Dependence of the scattering efficiency Q on the dimensionless size parameter kR_2, is depicted in Figs. 1. It is assumed that refractive indexes of the surrounding medium and the isotropic core are equal, $n = n_2$. Clearly, the scattering cross section is strongly affected by the type of anisotropy characterizing by the unit vector $\hat{\mathbf{n}}$. Fig. 1 shows that the structure $\hat{\mathbf{n}} = \hat{\boldsymbol{\varphi}}$ has the largest value of Q at small kR_2, whereas the least scattering efficiency corresponds to the configuration with $\hat{\mathbf{n}} = \hat{\boldsymbol{\vartheta}}$. We found that the helical structure $\hat{\mathbf{n}} = \hat{\boldsymbol{\varphi}}$ remains the most efficient scatterer with increase in the anisotropy parameter u_1 for sufficiently small particle size. On the other hand, Fig. 1 reveals that the configuration $\hat{\mathbf{n}} = \hat{\boldsymbol{\vartheta}}$ becomes the most efficient scatterer as size of the scatterer increases.

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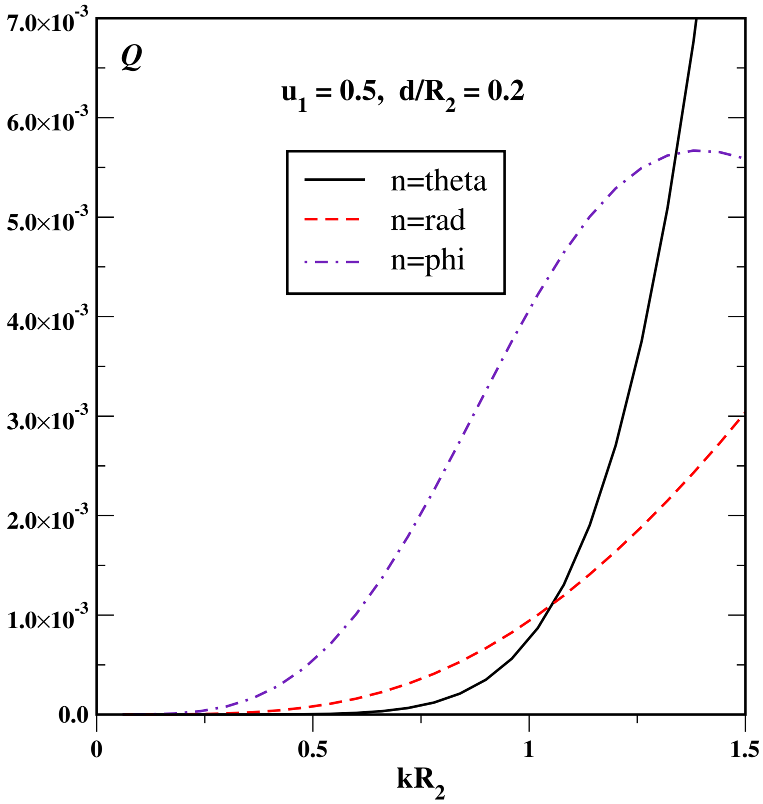

FIGURE 1: Dependence of Q on the size parameter kR_2 at $u_1 = 0.5$ and $d/R_2 = 0.2$ for different types of anisotropy. It is shown that the scattering efficiency reveals crossover behaviour in this range of size parameters.

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