Nonreciprocity and Spatial Dispersion in Bianisotropic Media

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Abstract

Definition of the notion of reaction in bianisotropic media is not so trivial. In this paper, we consider some important aspects of the physical admissibility to use the notion of the reaction as a "physical observable" in bianisotropic media. The questions also arise: For what kinds of the known bianisotropic media the reciprocity theorem is physically applicable? Based on what kind of the bianisotropic media, nonreciprocal microwave devices can be realized? We will show that a novel class of microwave bianisotropic materials - magnetostatically controlled bianisotropic materials (the MCBMs) - are "physically justified" materials. The Onsager-Casimir principle and the notion of reciprocity are applicable in this case. New nonreciprocal microwave devices based on the MCBMs can be realized.

1. Introduction

There are many attempts to generalize the reciprocity theorem for bianisotropic media. Gyrotropic (gyromagnetic or gyroelectric) media with non-symmetrical constitutive tensors caused by an applied dc magnetic field have been called "nonreciprocal" media because the usual reciprocity theorem [1] does not apply to them. Rumsey has introduced a quantity called the "reaction" and interpreted it as a "physical observable" [2]. This made it possible to obtain a modified reciprocity theorem based on the property of gyrotropic media that non-symmetrical constitutive tensors of permittivity or permeability are transposed by reversing the dc magnetic field [3]. Physically, the applicability of the reciprocity theorem for gyrotropic media is based on the time-reversal invariance which is described by the Onsager principle [4,5]. With formal introduction of the notion of reaction for bianisotropic media, one can formulate reciprocity conditions for medium parameters [6]. One can also extend the procedure used for a gyrotropic medium and consider the complementary [7], or the Lorentz-adjoint [8] bianisotropic medium which satisfies the reciprocity theorem. It was shown [8] that for the monochromatic electromagnetic field in the lossless bianisotropic medium, the time-reversed constitutive matrix may correspond to the Lorentz-adjoint constitutive matrix.

The questions, however, arise: Whether the time-reversal operations used for bianisotropic media are physically admissible? For what kinds of the known bianisotropic media the reciprocity theorem is physically applicable? One cannot *a priori* characterize bianisotropic media as reciprocal or non-reciprocal materials until a convincing analysis will show that microscopic properties "permit" time-reversal operations.

Recently, we have conceptualized a novel class of microwave bianisotropic materials based on a composition of ferrite magnetostatic wave (MSW) resonators with special-form surface metalizations - the magnetostatically controlled bianisotropic materials (MCBMs) [9]. The proposed MCBMs becomes not hypothetical materials after recent experimental results have verified the fact that ferrite magnetostatic-wave resonators with special-form surface metalizations exhibit properties of bianisotropic particles [10,11]. A very important aspect arises from the fundamental point of view. The MCBMs are *local temporally dispersive bianisotropic media* in comparison with helix or Ω-particle composites characterized as media with *nonlocal* properties [12,13].

2. Reciprocity Theorem for Bianisoropic Media

Reciprocity theorem shows that nonreciprocal bianisotropic media with constitutive parameters altered by reversing the dc magnetic field $\vec{H}_0$ are described as

$$\begin{aligned}\overset{\leftrightarrow}{\epsilon}(\omega,\vec{H}_0) &= \overset{\leftrightarrow}{\epsilon}^{T}(\omega,-\vec{H}_0),\ \overset{\leftrightarrow}{\mu}(\omega,\vec{H}_0)=\overset{\leftrightarrow}{\mu}^{T}(\omega,-\vec{H}_0),\\ \overset{\leftrightarrow}{\xi}(\omega,\vec{H}_0) &= -\overset{\leftrightarrow}{\zeta}^{T}(\omega,-\vec{H}_0),\ \overset{\leftrightarrow}{\zeta}(\omega,\vec{H}_0)=-\overset{\leftrightarrow}{\xi}^{T}(\omega,-\vec{H}_0),\end{aligned} \tag{1}$$

The microscopic aspects of relations (1) will be discussed in section 4.

Reciprocity theorem is an example of quadratic relations in electrodynamics of bianisotropic media. Energy relations are another important example of quadratic relations. It is clear that these two forms of quadratic relations should not be considered independently. In particular, the correct definition of the reaction in bianisotropic media should be made, taking into account the energy balance equation. The energy relations for bianisotropic media were carefully analyzed in [14] and also discussed in [13]. One can see that energetic relations for bianisotropic media cannot be considered just as an extension of the similar relations for anisotropic media. In bianisotropic media, variation of the energy should be realized due to both types of sources, both types of currents - the electric and magnetic currents.

3. Network Reciprocity

Let us consider now some general properties of waveguide junctions containing bianisotropic samples. It is not so difficult to show that for two sets of the fields due to sources a and b we can write

$$\begin{aligned}&\int\limits_S\left(\vec{H}_a\times\vec{E}_b-\vec{H}_b\times\vec{E}_a\right)\cdot\vec{n}\,dS-i\omega\int\limits_{V'}\Big[\vec{E}_b\cdot\left(\overset{\leftrightarrow}{\epsilon}\cdot\vec{E}_a\right)-\vec{E}_a\cdot\left(\overset{\leftrightarrow}{\epsilon}\cdot\vec{E}_b\right)+\\ +&\ \vec{H}_a\cdot\left(\overset{\leftrightarrow}{\mu}\cdot\vec{H}_b\right)-\vec{H}_b\cdot\left(\overset{\leftrightarrow}{\mu}\cdot\vec{H}_a\right)+\vec{E}_b\cdot\left(\overset{\leftrightarrow}{\xi}\cdot\vec{H}_a\right)+\vec{H}_a\cdot\left(\overset{\leftrightarrow}{\zeta}\cdot\vec{E}_b\right)-\\ -&\ \vec{E}_a\cdot\left(\overset{\leftrightarrow}{\xi}\cdot\vec{H}_b\right)-\vec{H}_b\cdot\left(\overset{\leftrightarrow}{\zeta}\cdot\vec{E}_a\right)\Big]\,dV=\sum_{p=1}^{N}\int\limits_{S_p}\left(\vec{H}_a^{(p)}\times\vec{E}_b^{(p)}-\vec{H}_b^{(p)}\times\vec{E}_a^{(p)}\right)\cdot\vec{n}^{(p)}\,ds=\\ =&\ i\omega\int\limits_{V'}\Big[2\vec{E}_b\cdot\left(\overset{\leftrightarrow}{\epsilon}_{as}\cdot\vec{E}_a\right)-2\vec{H}_b\cdot\left(\overset{\leftrightarrow}{\mu}_{as}\cdot\vec{H}_a\right)+\vec{E}_b\cdot\left(\overset{\leftrightarrow}{A}\cdot\vec{H}_a\right)-\vec{E}_a\cdot\left(\overset{\leftrightarrow}{A}\cdot\vec{H}_b\right)\Big]\,dV\equiv K\end{aligned} \tag{2}$$

where S is a surface that restricts the volume V, $\vec{n}$ is the unit vector along the external normal to the surface S, V' is a part of a volume V filled by the bianisotropic medium. We chose volume V as the volume restricted by a joint of several waveguides and cross sections of these waveguides and supposed that sources are placed beyond the volume V. We also took into account that a surface integral over metallic walls is equal to zero.

In Expr. (2) p is a number of a port, S_p and $\vec{n}^{(p)}$ are, respectively, a cross-section of port p and the unit normal vector to this cross-section, $\overset{\leftrightarrow}{\epsilon}_{as}$ and $\overset{\leftrightarrow}{\mu}_{as}$ are antisymmetric parts of tensors $\overset{\leftrightarrow}{\epsilon}$ and $\overset{\leftrightarrow}{\mu}$, respectively, and $\overset{\leftrightarrow}{A}\equiv\overset{\leftrightarrow}{\xi}+\overset{\leftrightarrow}{\zeta}^{T}$. Besides a trivial case of $\overset{\leftrightarrow}{\epsilon}_{as}=\overset{\leftrightarrow}{\mu}_{as}=\overset{\leftrightarrow}{A}=0$, the quantity

K may become equal to zero for some particular cases of the field structure and the geometry of a problem when $\overleftrightarrow{\epsilon}_{as} \neq 0, \overleftrightarrow{\mu}_{as} \neq 0$ and $\overleftrightarrow{A} \neq 0$. We suppose that, in a general case, $K \neq 0$ for $\overleftrightarrow{\epsilon}_{as} \neq 0, \overleftrightarrow{\mu}_{as} \neq 0$, and $\overleftrightarrow{A} \neq 0$.

Let all ports, besides ports p and q, be short-circuited. In this case, the left-hand side of (2) may be rewritten in terms of normalized voltages applied to ports p and q and in terms of admittance matrix [Y] [15]. As a result, we have for (2):

$$\left(V_a^{(p)} V_b^{(q)} - V_b^{(p)} V_a^{(q)}\right) \left(Y_{pq} - Y_{qp}\right) = K \tag{3}$$

Normalized voltages $V_a^{(p)}, V_b^{(q)}, V_b^{(p)}$ and $V_a^{(q)}$ are arbitrary. Therefore, the nonreciprocal difference for parameters of the admittance matrix, $Y_{pq} - Y_{qp}$, is defined by integral K.

Taking into account correlation between the admittance matrix [Y] and scattering matrix [S] [15], one can rewrite (3) as follows:

$$Q \left(S_{pq} - S_{qp}\right) = K \ , \tag{4}$$

where term Q is a coefficient determined by amplitudes of the fields. Since coefficient Q is arbitrary, nonreciprocal difference for parameters of the scattering matrix, $S_{pq} - S_{qp}$, is defined by integral K.

Now the main question arises: When devices with integral $K \neq 0$ can be really characterized as nonreciprocal devices? Let us suppose that we have an Y-circulator constructed as a three-port waveguide junction with enclosed a sample of a bianisotropic material. When in (4) $K \neq 0$, one can realize a matched nonreciprocal three-port junction [15,16]. In a case of ferrite devices, we have an example of the magnetic group of symmetry of a system: "a waveguide junction + dc magnetic field" [17]. One of the main principle of the known nonreciprocal devices [3] sounds as: simultaneous exchange of ports and dc magnetic field direction does not alter the transmission properties. This principle follows from relation (4) if K alters its sign with alteration of dc magnetic field direction. The sign of K, in turn, will be changed when tensors, $\overleftrightarrow{\epsilon}_{as}, \overleftrightarrow{\mu}_{as}$ and $\overleftrightarrow{A}$ change their signs for opposite dc magnetic field direction. It is evident that we satisfy these conditions when relations (1) take place. Is one able to realize an Y-circulation without satisfaction to the principle of nonreciprocal devices mentioned above? When an answer to this question is positive, one will have *enantiomorphic devices*: left- or right-handed Y-circulators. The MCBMs give an example of nonreciprocal bianisotropic materials that allow to realize devices which satisfy the principle of nonreciprocal devices mentioned above. In comparison with the known gyrotropic media where the effect of nonreciprocity is due to the time-reversal invariance of microscopic equations of motion in ferrites or plasmas, in the MCBMs we have the effect of *nonreciprocal magnetoelectric coupling* between electric and magnetic dipoles in every bianisotropic particle. This, our standpoints, is based on consideration of microscopic properties of the MCBM-particles [13,18].

4. Discussion and Conclusion

To ensure "physicality", we have to rely on the generalized principle of kinetic coefficient symmetry for bianisotropic media - the Onsager-Casimir principle. It was shown in [13,18] that, in a general case, reciprocity relations (1) may not correspond to dynamical constitutive symmetry obtained from Onsager-Casimir principle. The answer to the question: Whether the *microscopic properties* of a bianisotropic medium "permit" the time-reversal operations, should be found from an analysis of the symmetry of the dynamical processes in the MCBM-particles under the time-reverse operations. *Nonsymmetry of the constitutive matrix is a measure of nonreciprocity in the MCBMs.* In accordance with (4), we can characterize a measure of nonreciprocity for a bianisotropic medium by the matrix parameter $\overset{\leftrightarrow}{A} = \overset{\leftrightarrow}{\xi} + \overset{\leftrightarrow}{\xi}^{T}$. Our analysis of the network reciprocity shows that the MCBM-devices should satisfy one of the main principle of the known nonreciprocal devices: simultaneous exchange of ports and dc magnetic field direction does not alter the transmission properties.

Can chiroferrites (chiroplasmas) be considered as nonreciprocal bianisotropic materials? Really, in this case, one has a composite medium based on the gyrotropic host material. Constitutive parameters of such media should be dependent on the external dc magnetic field. But, the question is still open: How can one consider the time-reversal operation in media with a lack of symmetry (chiral inclusions)? Any physical justifications based on dynamical constitutive symmetry (the Onsager-Casimir principle) are not applicable in this case.

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