Point Sources of Magnetoelectric Fields

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Abstract

In this paper, we show that the unified quasistatic magnetoelectric fields (QME fields) originated by point sources - the quasistatic magnetoelectric particles (QME particles) - can exist when symmetry properties of these fields are distinguished from that of the electromagnetic fields. The physical ground for QME particles can be found in small ferromagnetic resonators where short-wavelength (so-called magnetostatic) oscillations take place. The question about QME particles and fields arises in such a topical subject as artificial bianisotropic materials.

1. Introduction

The question about coupling of electric and magnetic polarizations arises in problems of artificial chiral and bianisotropic media [1,2]. It is supposed that there exists a possibility to describe properties of media with intrinsic coupling between the electric and magnetic polarizations by phenomenological constitutive relations with further use these constitutive relations in Maxwell's equations. Fundamental contradictions in such an approach can be perceived, however. Some of these contradictions concerning nonlocal properties of bianisotropic composites based on small helices and so-called omega-particles, we have recently discussed [3-5]. Is it possible to have artificial bianisotropic media with local properties? One can suppose that these composite materials should be based on *point* bianisotropic (magnetoelectric) particles that can be considered, by a simple model, as small coupled electric and magnetic dipoles. However, the question suggests itself: Can one consider (classical electrodynamically) two small, i.e. quasistatic (with sizes much less than the electromagnetic wavelength) coupled electric and magnetic dipoles as *point sources of the electromagnetic field*? To answer this question, the following aspects should be taken into account: (a) neither Lorentz nor Coulomb gauges [6] can be used to describe the fields of such particles and (b) mechanical interaction between two such particles cannot be described by the Lorentz force since potential energy of every particle is characterized not only by electric and magnetic energies, but also by energy of internal coupling between the electric and magnetic dipoles. Based on these aspects and also taking into account the fact that in a small region (much less than the electromagnetic wavelength) of sourceless free space a character of the unified field is physically undefined by the Maxwell equations, we should come to conclusion that *the unified quasistatic magnetoelectric fields (QME fields) originated by point sources - the quasistatic magnetoelectric particles (QME particles) - can exist when symmetry properties of these fields are distinguished from that of the electromagnetic fields.*

The physical ground for QME particles can, in particular, be found in small ferromagnetic resonant specimens where short-wavelength (so-called magnetostatic) oscillations take place [7]. In a case of small normally magnetized ferrite disks placed into a region of the uniform rf magnetic field, a long series of oscillating magnetostatic modes were observed experimentally [8,9]. Recently, we have shown that these oscillations can be characterized by a discrete spectrum of energy levels [10]. The QME particles one can realize based on ferrite resonators with special-form surface electrodes. In this case, magnetostatic oscillations in a ferrite body are accompanied with short-wavelength surface-electric-charge oscillations on a metallic electrode [3,11]. Recently, the first experimental evidence for this magnetoelectric coupling in small ferrite resonators with special-form surface electrodes has been obtained [12,13].

2. Symmetry Properties of Ferrite QME Particles

Based on a semi-classical model, one can illustrate some symmetry properties of ferrite QME particles. These particles can conventionally be described as a pair of two coupled dipoles: an electric dipole with moment $\vec{p^e}$ and a magnetic dipole with moment $\vec{p^m}$. These two dipoles are mutually perpendicular and the bias magnetic field $\vec{H_0}$ is oriented perpendicular to the plane of $\vec{p^e}$, $\vec{p^m}$ vectors. So, one has a triple of mutually perpendicular vectors: $\vec{p^e}$, $\vec{p^m}$ and $\vec{H_0}$. As we discussed in [5], the PT invariance (the time-reversal operation T combined with the parity P) does not hold in this model of one polar ($\vec{p^e}$) and two axial ($\vec{p^m}$ and $\vec{H_0}$) vectors. But, the CPT invariance (when the charge conjugation C changes the sign of vector $\vec{p^e}$) takes place.

It should be clearly understood that in this our semi-classical model of a QME particle, the charge conjugation C does not mean interconversion of electrons and positions in the magnetic processes (of the atomic scales) in a ferrite. As an important fact, it is discussed in this paper that the problem of the charge conjugation has to be considered in connection with the *sign of energy eigenstates* of oscillations in a QME particle. To clarify the problem, we should analyze the "microscopic properties" of magnetostatic oscillations in ferrite QME particles.

3. Energy Eigenstates Magnetostatic Oscillations

The average magnetostatic energy of magnetostatic oscillations in a "pure" (without surface metallic electrodes) normally magnetized ferrite disk, having a small ratio of thickness h to radius a can be described as [10]:

$$\begin{aligned} \bar{W} &= \frac{w\mu_0}{4} \int_s \left[X^{(\mathcal{D})} \left(\int_{-\infty}^{0} \psi\psi^* dz + \int_{h}^{\infty} \psi\psi^* dz \right) + \right. \\ &+ \left. X^{(F)} \int_0^h \psi\psi^* dz \right] ds \end{aligned} \tag{1}$$

where ψ is the magnetostatic potential, $X^{(\mathcal{D})}$ and $X^{(F)}$ are coefficients characterizing wave processes, respectively, in dielectric $\mathcal{D}$ $(z \le 0,\ z \ge h)$ and ferrite F $(0 \le z \le h)$ regions. Coefficients X are found from the equations

$$-\frac{i}{X} \nabla_{\parallel}^2 \psi = \frac{\partial\psi}{\partial t} \tag{2}$$

written for every region. Here $\nabla_{\parallel}^2$ is a longitudinal part of Laplace operator.

We suppose that in a ferrite disk resonator, magnetostatic potential ψ can be represented as [10]

$$\psi = \sum_{p,q} A_{pq}\ \tilde{\xi}_p\ (z)\ \tilde{\varphi}_q\ (\rho, \alpha) \tag{3}$$

For "in-plane" resonant mode $\tilde{\varphi}_q$ (and p "thickness" mode), one has an operator equation for normalized energy of magnetosttic oscillations [10]

$$\hat{F}_\perp \tilde{\varphi}_q = E_{pq} \tilde{\varphi}_q \tag{4}$$

Differential operator $\hat{F}_\perp$ is defined as

$$\hat{F}_\perp = g\frac{\mu_0}{4}\nabla_\perp^2 \tag{5}$$

where $\nabla_\perp^2$ is two-dimensional ("in-plane") Laplace operator, g is unit dimensional co-efficient. The property of energy orthonormality for magnetostatic oscillations is described as

$$(E_{pq} - E_{pq'})\int_Q \tilde{\varphi}_q\tilde{\varphi}^*_{q'}ds = 0 \tag{6}$$

where Q is a square of "in-plane" cross section of an open ferrite disk including radius regions $Q_F(\rho \leq a)$ and $Q_\mathcal{D}(a \leq \rho < \infty)$.

4. Effect of the External Fields

We can write the Hamiltonian taking into account an interaction of the magnetoelectric particle with the exciting electric $\vec{\varepsilon}$ and magnetic $\vec{\mathcal{H}}$ fields. For a disk with unit thickness, the Hamiltonian has a form [14]

$$\hat{H} = \hat{F}_\perp + \hat{F}'_\perp + \frac{A}{b}\hat{\vec{l}}\,\hat{\vec{p^e}} + \mathcal{D}\hat{\vec{p^e}}\,\vec{\varepsilon} \tag{7}$$

where $\mathcal{D}$ is a constant that can be positive or negative, $\hat{F}'_\perp$ is the term of extra energy of the particle due to the external magnetic field.

Operator $\hat{F}'_\perp$ can be written as

$$\hat{F}'_\perp = \vec{\mathcal{H}}\,\hat{\vec{m}}_\perp \tag{8}$$

Here $\hat{\vec{m}}_\perp$ is the operator of the "in-plane" alternative magnetization. Operators $\hat{F}_\perp$ and $\hat{m}_\perp$ *do not commute* one with another. For a ferrite disk resonator placed in a tangential rf magnetic field, we have energy perturbations similar to those one has for atom placed in an external electric field (the Stark effect) [15]: the second-order effect of a split of energy levels in the external rf magnetic field. The energy split ΔE_n of energy level E_n is proportional to

$$\Delta E_n \sim \alpha_{ij}^{(n)}\mathcal{H}_i\mathcal{H}_j \tag{9}$$

where $\alpha_{ij}^{(n)}$ is the tensor of magnetic polarization ("in-plane") of a ferrite disk. For every energy level, an average magnetic moment of a ferrite disk is

$$(p^m)_i = \alpha_{ij}\mathcal{H}_j \tag{10}$$

Unlike the average magnetic moment, electric moments of a ferrite disk are not equal to zero in oscillation eigenstates. An interaction between the external rf electric field and a ferrite disk is similar to an interaction between the electrically neutral particles (neutron, atom) with ("magnetic") spin and external alternative magnetic field [15].

The particle can have potential energy in the external electric $\vec{\varepsilon}$ field. Similarly, the particle can possess potential energy in the external magnetic $\vec{\mathcal{H}}$ field. At the same time, because of the effect of internal magnetoelectric coupling, the particle should have potential energy in the combined $\vec{\varepsilon} + \vec{\mathcal{H}}$ field. The structure of this combined external $\vec{\varepsilon} + \vec{\mathcal{H}}$ field is not just a superposition of the $\vec{\varepsilon}$ and $\vec{\mathcal{H}}$ fields. It is clear that the $\vec{\varepsilon}$ and $\vec{\mathcal{H}}$ components of this (combined) field should be in a certain phase correlating to provide a maximum of potential energy. Because of this correlation we have to talk about the *unified* - quasistatic magnetoelectric

(QME) - field. If we suppose that potential energy due to the effect of magnetoelectric coupling is a small part of summarized potential energy that a particle has in separated $\vec{\varepsilon}$ and $\vec{\mathcal{H}}$ field, the total potential energy E_{total} can be represented as a sum of three components:

$$E_{total} = E_{\varepsilon} + E_{\mathcal{H}} + E' \,, \tag{11}$$

where E_{ε} is the potential energy of a particle due to external electric field, $E_{\mathcal{H}}$ is the potential energy of a particle due to external magnetic field, and E' is the potential energy due to a combined effect of action of two $\left(\vec{\varepsilon} \text{ and } \vec{\mathcal{H}}\right)$ fields

Conclusion

Based on "microscopic properties" of magnetostatic oscillations in ferrite disk resonators, we have shown, in this paper, that such resonators can be considered as QME particles: the point sources of unified QME fields. The symmetry properties of these fields are distinguished from that of the electromagnetic fields.

Magnetostatic oscillations in a ferrite disk resonator can be characterized by eigen angular momentum of magnetostaic oscillations with the so-called "electric spin" and the eigen electric moment. Similarly to neutrino (where spin is not separated from orbit moment), in our case, "electric spin" is also not separated from the angular momentum operator.

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