Localized Electromagnetic Fields in Complex Media and Free Space

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Abstract

The presented exact solutions of homogeneous Maxwell's equations in complex media and free space describe fields having a rather involved curl structure and a very small—about several wave lengthes of composing plane waves—and clearly defined core region with maximum intensity of field oscillations. In a given Lorentz frame L, a set of the obtained exact time–harmonic solutions of the free–space homogeneous Maxwell equations consists of three subsets—termed "storms", "whirls", and "tornadoes" for the sake of brevity—for which time average energy flux is identically zero at all points, azimuthal, and spiral, respectively. In any other Lorentz frame L', they will be observed as a kind of electromagnetic missiles moving without dispersing at speed $V < c$. The solutions which describe finite–energy evolving electromagnetic storms, whirls, tornadoes are also presented.

1. Introduction

In the beginning of eighties, Brittingham [1] proposed the problem of searching for specific electromagnetic waves—focus wave modes—having a three–dimensional pulse structure, being nondispersive for all time, and moving at light velocity in straight lines. A number of packet–like solutions have been presented [1, 2, 3]. However, it seems [3, 4, 5], finite–energy focus wave modes can not exist without sources. In 1985, Wu introduced [5] a conception of electromagnetic missiles moving at light velocity and having a very slow rate of decrease with distance. Because of these properties, such missiles have important possible applications [5].

In Ref. [6], vector plane–wave superpositions defined by a given set of orthonormal scalar functions on a two–dimensional or three–dimensional manifold—beam manifold $\mathcal{B}$—are treated. The proposed approach makes it possible to compose a set of orthonormal beams, normalized to either the energy flux through a given plane σ_0 with unit normal $\mathbf{q}$ (beams with two–dimensional $\mathcal{B}$) or to the total energy transmitted through this plane (beams with three–dimensional $\mathcal{B}$), as well as some other specific exact solutions of wave equations such as three–dimensional standing waves, moving and evolving whirls. This approach can be applied to any linear field, such as electromagnetic waves in free space, isotropic, anisotropic, and bianisotropic media, elastic waves in isotropic and anisotropic media, sound waves, etc. By way of illustration, some specific families of exact solutions of the homogeneous Maxwell equations, describing localized electromagnetic fields in free space, are obtained in Refs. [6, 7, 8]. In this paper, we present some new types of such localized electromagnetic fields in complex media and free space.

2. Time–Harmonic Localized Fields Defined by the Spherical Harmonics

In this paper, we treat time–harmonic electromagnetic fields in linear complex media or free space, defined by the spherical harmonics as

$$\mathbf{W}_j^s(\mathbf{r},t) = e^{-i\omega t}\int_0^{2\pi} d\varphi \int_{\theta_1}^{\theta_2} e^{i\mathbf{r}\cdot\mathbf{k}(\theta,\varphi)} Y_j^s(\theta,\varphi)\nu(\theta,\varphi)\mathbf{W}(\theta,\varphi)\sin\theta d\theta, \tag{1}$$

where $\mathbf{W} = \mathrm{col}(\mathbf{E},\mathbf{B})$ specifies the polarization of the eigenwave with the wave vector $\mathbf{k}$. To compose these beams, it is necessary first to calculate parameters of eigenwaves. The corresponding relations for electromagnetic waves in a general bianisotropic medium are presented in Ref. [6].

There are two main ways to set the beam base, i.e., to specify the functions $\mathbf{k} = \mathbf{k}(\theta,\varphi)$ and $\mathbf{W} = \mathbf{W}(\theta,\varphi)$. One can set first the unit wave normals of these eigenwaves by a function $\hat{\mathbf{k}} = \hat{\mathbf{k}}(\theta,\varphi)$. In particular, one can set the angular spectrum of plane waves by

$$\hat{\mathbf{k}} = \mathbf{k}/k = \sin\theta'(\mathbf{e}_1\cos\varphi + \mathbf{e}_2\sin\varphi) + \mathbf{e}_3\cos\theta, \tag{2}$$

where $\theta' = \kappa_0\theta$, and κ_0 is some real parameter. Then, one has to calculate the refractive indices $n_j(\theta,\varphi) = n_j(\hat{\mathbf{k}}(\theta,\varphi))$ of all isonormal waves and, by choosing some branch $n_j(\theta,\varphi)$, to specify the wave vector function $\mathbf{k}(b) = (\omega/c)n_j(\theta,\varphi)\hat{\mathbf{k}}(\theta,\varphi)$ and the amplitude function $\mathbf{W}(\theta,\varphi) = \mathrm{col}(\mathbf{E}(\theta,\varphi),\mathbf{B}(\theta,\varphi))$ as well. The alternative is to set first the tangenial components of wave vectors by a real vector function $\mathbf{t} = \mathbf{t}(\theta,\varphi)$. Then, the normal component $\xi_j(\theta,\varphi) = \xi_j(\mathbf{t}(\theta,\varphi))$ of $\mathbf{k}(\theta,\varphi) = \mathbf{t}(\theta,\varphi) + \xi_j(\theta,\varphi)\mathbf{q}$ is chosen from the roots of a quartic equation [6]. In addition to the parameters of eigenwaves themselves in the medium under study, there are three parameters defining the properties of the presented beams: θ_1, θ_2, and κ_0. By setting these parameters in various ways, one can compose various localized fields with very interesting properties. Let us illustrate this on the case of localized fields in free space.

In free space, the integral representations of the localized fields under consideration and the orthonormal beams, presented in Ref. [9], differ only by the values of the integration limits in Eq. (1). In both cases, there are beams with two independent polarization states—E_M and E_A beams.

Let us consider time–harmonic fields $\mathbf{W}_j^s$ [Eq. (1)] with $\theta_1 = 0$, $\pi/2 \le \theta_2 \le \pi$, and $\kappa_0 = 1$, i.e., with $\theta' = \theta$. These fields are composed of plane waves propagating in a solid angle $\Omega \in [2\pi, 4\pi]$. For the sake of simplicity, we assume that the beam state function $\nu = \nu(\theta,\varphi)$ reduces to a constant. A set of these exact time–harmonic solutions of the free-space homogeneous Maxwell equations consists of three subsets—termed "storms", "whirls", and "tornadoes" for brevity—for which time average energy flux is identically zero at all points, azimuthal, and spiral, respectively.

Let us first set $\theta_2 = \pi$. Then, the fields under consideration are composed from plane waves of all possible propagation directions, i.e., $\Omega = 4\pi$. They are in effect three–dimensional standing waves with rather involved structures of interrelated electric and magnetic fields. Beams with $s \neq 0$ are essentially electromagnetic whirls with azimuthal energy fluxes. For E_A and B_A electromagnetic storms, both of which are defined by the zonal spherical harmonics ($s = 0$), the time average Poynting vector $\mathbf{S}$ is vanishing at all points. The electric field vector $\mathbf{E}$ of E_A storms has the only—azimuthal—component, whereas the azimuthal component of the magnetic field vector $\mathbf{B}$ is everywhere zero. The opposite situation occurs with B_A storms.

The spherical harmonics with $s \neq 0$ define electromagnetic whirls for which the time average Poynting vector $\mathbf{S}$ has the only nonvanishing—azimuthal—component. This component, as well as the energy densities w_e and w_m of the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields, is independent of the azimuthal angle ψ. The whirls with $j > s \ge 1$ have two major domains—above and below the equatorial plane—with large energy fluxes. The whirls, defined by the sectorial harmonics ($j = s \geq 1$), have only one such domain, and the energy flux peaks in the equatorial plane.

Let us now consider time–harmonic fields $\mathbf{W}_j^s$ [Eq. (1)] with $\theta_1 = 0$, $\pi/2 < \theta_2 < \pi$, and $\kappa_0 = 1$, i.e., with $\theta' = \theta$, and $2\pi < \Omega < 4\pi$. As before, we assume that the beam state function $\nu = \nu(\theta, \varphi)$ reduces to a constant. In this case, the field also is highly localized, but the normal and the radial components of time average Poynting's vector $\mathbf{S}$ are not vanishing. As a result, lines of energy flux become spiral, provided that $s \neq 0$. We refer to such specific localized fields with spiral energy fluxes as electromagnetic tornadoes. Their lines of energy flux closely resemble spirals, and as θ_2 tends to π, the step of these spirals decreases. For the fields defined by the zonal spherical harmonics ($s = 0$), the lines of energy flux lie in meridional planes.

3. Evolving Storms, Whirls, and Tornadoes

Although, in many cases, the presented time–harmonic solutions may provide satisfactory models of real physical fields, more accurate models can be obtained by integrating these solutions with respect to frequency. In particular, some solutions which describe quasimonochromatic electromagnetic beams are obtained in Ref. [6].

Let us consider localized fields $\breve{\mathbf{W}}_j^s(\mathbf{r}, t)$ with three–dimensional beam manifold $\mathcal{B}_3 = \mathcal{B} \times [\omega_-, \omega_+]$, related with $\mathbf{W}_j^s(\mathbf{r}, t)$ [Eq. (1)] as

$$\breve{\mathbf{W}}_j^s(\mathbf{r}, t) = \frac{1}{2\Delta\omega} \int_{\omega_-}^{\omega_+} \mathbf{W}_j^s(\mathbf{r}, t) d\omega, \tag{3}$$

where $\Delta\omega = (\omega_+ - \omega_-)/2$. In the case of quasimonochromatic beams, $\Delta\omega \ll \omega$. For the beams under consideration, the amplitude function $\mathbf{W}(\theta, \varphi)$ is frequency independent. If the beam state function $\nu(\theta, \varphi)$ also is frequency independent, or its frequency dependence is negligibly small, we have

$$\begin{aligned} \breve{\mathbf{W}}_j^s(\mathbf{r}, t) = \quad & e^{-i\omega t} \int_0^{2\pi} d\varphi \int_{\theta_1}^{\theta_2} e^{i\mathbf{r}\cdot\mathbf{k}(\theta,\varphi)} Y_j^s(\theta, \varphi) j_0 \left[p_0 \left(\mathbf{r} \cdot \mathbf{k}(\theta, \varphi) - \omega t\right)\right] \\ & \times \nu(\theta, \varphi) \mathbf{W}(\theta, \varphi) \sin\theta d\theta, \end{aligned} \tag{4}$$

where $\omega = (\omega_+ + \omega_-)/2$ and $p_0 = \Delta\omega/\omega$. In free space, this field is composed of plane wave packets radially moving with the light velocity c.

Therefore, the field under considerarion is essentially an evolving whirl in the neighbourhood of the point $\mathbf{r} = 0$. It varies in intensity as different "peaks and valleys" reach the core region. At $-\pi/\Delta\omega < t < \pi/\Delta\omega$, the main peak passes through this region, and the whirl reaches its absolute maximum intensity at $t = 0$. At this moment, its field structure is very close to the structure of the corresponding time–harmonic whirl. In particular, lines of energy flux are circular for both whirls. At $-\pi/\Delta\omega < t < 0$ and $0 < t < \pi/\Delta\omega$, the energy flux lines of the evolving whirls are convergent and divergent, respectively.

On the whole, the evolution of the field can be described as follows. When $t \to \pm\infty$, the field tends to zero at all points $\mathbf{r}$. Therefore, the solution $\breve{\mathbf{W}}_j^s(\mathbf{r}, t)$ [Eq. (4)] describes initiation and evolution of a whirl, which originates at the infinity at $t = -\infty$ as infinitely small converging wave propagating with the light velocity c. At $t \ll -\pi/\Delta\omega$, there is a very small converging wave with maximum peak at the distance $r = -ct$. During all this time, there is also a weak whirl in the neighbourhood of the point $\mathbf{r} = 0$. It passes through maximums and minimums of activity, gradually gaining in intensity as t tends to zero.

The total field can be described as the superposition of converging and expanding waves with ever changing proportion. At $t > 0$, the whirl, still passing through maximums and minimums of activity, gradually transforms into expanding wave, which vanishes in the infinity as $t \to +\infty$.

It is significant that the evolving storms, whirls, and tornadoes have finite total energy.

4. Conclusion

In this paper, new families of exact solutions of the homogeneous Maxwell equations in complex media and free space, obtained on the basis of angular–spectrum representations, are presented. These families of solutions specify three–dimensionally localized electromagnetic fields having a rather involved curl structure and a very small—about several wave lengthes of composing plane waves—and clearly defined core region with maximum intensity of field oscillations. Outside of the core, the intensity of oscillations rapidly decrease in all directions. In complex media, these fields provide a unique global description of the medium under study, which is supplementary to the eigenwave description. Whereas each eigenwave specifies the properties of the medium for one particular direction of propagation, the field value of a three–dimensional standing wave in any point is defined by all eigenwaves. Moreover, even in free space such waves possess very interesting properties. In a given Lorentz frame L, a set of obtained time–harmonic free–space solutions consists of three subsets — termed "storms", "whirls", and "tornadoes" for brevity sake — for which time average energy flux is identically zero at all points, azimuthal, and spiral, respectively. In any other Lorentz frame L', they will be observed as a kind of electromagnetic missiles moving without dispersing at speed $V < c$. Solutions which describe, in the frame L, finite–energy quasi–monochromatic evolving electromagnetic storms, whirls, tornadoes, and correspondingly, in the frame L', various types of moving and evolving missiles, are also obtained. The intrinsic tensor technique in Minkowski space [10] provides a convenient means for investigating all these new types of waves. Their properties are illustrated in graphic form.

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