The Application of Orthonormal Electromagnetic Beams to Characterizing Complex Media

G. N. Borzdov

Department of Theoretical Physics, Belarusian State University
Fr. Skaryny avenue 4, 220050 Minsk, Belarus
Fax: + 375–17–226 0530; email: borzdov@phys.bsu.unibel.by

Abstract

We present an approach to characterizing complex media, based on the use of new families of orthonormal electromagnetic beams. Each familly consists of orthonormal exact solutions of Maxwell's equations, which differ fundamentally from the well–known approximate solutions — the Hermite–Gaussian and Laguerre–Gaussian beams. A promising type of such orthonormal beams—beams defined by the spherical harmonics—is discussed. The proposed approach makes it possible to use beams focused into a small spot on the sample surface.

1. Introduction

The free–space techniques for characterizing complex media are based on the use of the plane–wave approximation of the incident beam. Computer modelling [1, 2, 3] of the free–space techniques [1, 2, 3, 4], based on the covariant impedance methods, has shown that these techniques make it possible to extract all material parameters of an anisotropic, chiral, or general bianisotropic medium, provided that the reflection and transmission coefficients of planar samples under normal and oblique incidence of plane harmonic waves are measured with sufficient accuracy. However, this requires a rather complicated measurement setup, and in many cases the plane–wave approximation of beams in use proves to be inadequate, especially for thick samples.

The technique presented in Refs. [5, 6] makes it possible to compose a set of orthonormal beams in a complex medium or free space, normalized to the energy flux through a given plane. They can be used to generalize the free–space techniques [1, 2, 3, 4] for characterizing complex media, developed for the case of plane incident waves, to the case of incident beams. A promising type of such orthonormal beams—beams defined by the spherical harmonics—is introduced in Ref. [7]. In this paper, we discuss the properties and applications of these beams in more detail.

2. Beams Defined by the Spherical Harmonics

In this paper, we consider electromagnetic fields in free space of the form [7]

$$\mathbf{W}^s_j(\mathbf{r},t) = e^{-i\omega t}\int_0^{2\pi} d\varphi \int_{\theta_1}^{\theta_2} e^{i\mathbf{r}\cdot\mathbf{k}(\theta,\varphi)} Y^s_j(\theta,\varphi)\nu(\theta,\varphi)\mathbf{W}(\theta,\varphi)\sin\theta d\theta. \tag{1}$$

They are defined by the spherical harmonics

$$Y^m_l(\theta,\varphi) = N_{lm}P^{|m|}_l(\cos\theta)e^{im\varphi}, \tag{2}$$

where

$$N_{lm} = \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}}, \tag{3}$$

and $P_l^m(\cos\theta)$ and $j_l(kr)$ are the spherical Legendre and Bessel functions. The spherical harmonics $Y_l^m(\theta,\varphi)$ satisfy the relations

$$\begin{aligned}\langle Y_l^m|Y_{l'}^{m'}\rangle \equiv & \int_0^{2\pi} d\varphi \int_0^{\pi} Y_l^{m*}(\theta,\varphi)Y_{l'}^{m'}(\theta,\varphi)\sin\theta d\theta \\ & = \delta_{ll'}\delta_{mm'}. \end{aligned} \tag{4}$$

Hence, for the beams under consideration (see also Ref. [8]), $\mathcal{B}_u$ is a unit sphere ($\mathcal{B}_u = S^2$), the beam manifold $\mathcal{B} \subset \mathcal{B}_u$ is its zone with $\theta \in [\theta_1, \theta_2]$ and $\varphi \in [0, 2\pi]$; and $d\mathcal{B} = \sin\theta d\theta d\varphi$.

To compose electromagnetic beams in free space, it is convinient to set $\mathbf{W} = \mathrm{col}(\mathbf{E},\mathbf{B}) = \mathrm{col}(\mathbf{E},\mathbf{H})$. For a time–harmonic field, the component $S_3 = \mathbf{e}_3 \cdot \mathbf{S}$ of the time average Poynting vector $\mathbf{S}$ can be written as

$$S_3 = \frac{c}{16\pi}\mathbf{e}_3 \cdot (\mathbf{E}^* \times \mathbf{H} + \mathbf{E} \times \mathbf{H}^*) = \mathbf{W}^\dagger Q \mathbf{W}, \tag{5}$$

where

$$Q = \frac{c}{16\pi}\begin{pmatrix} 0 & \mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_2 \otimes \mathbf{e}_1 \\ \mathbf{e}_2 \otimes \mathbf{e}_1 - \mathbf{e}_1 \otimes \mathbf{e}_2 & 0 \end{pmatrix}, \tag{6}$$

and $\otimes$ is the tensor product. Therefore, for the electromagnetic beams $\mathbf{W}_j^s$, the condition $\langle \mathbf{W}_j^s|Q|\mathbf{W}_j^s\rangle = N_Q$ is in fact the normalization to the beam energy flux N_Q through the plane σ_0 normal to $\mathbf{q} = \mathbf{e}_3$:

$$\langle \mathbf{W}_j^s|Q|\mathbf{W}_j^s\rangle = \int_{\sigma_0} S_3 d\sigma_0 = N_Q. \tag{7}$$

Each family of the fields under consideration is described by functions which have integral expansions in plane waves with wave normals lying in the same given solid angle Ω. In particular, one can set the angular spectrum of plane waves by

$$\hat{\mathbf{k}} = \hat{\mathbf{k}}(\theta,\varphi) \equiv \hat{\mathbf{k}}[\theta'(\theta,\varphi), \varphi'(\theta,\varphi)], \tag{8}$$

where

$$\hat{\mathbf{k}} = \mathbf{k}/k = \sin\theta'(\mathbf{e}_1\cos\varphi' + \mathbf{e}_2\sin\varphi') + \mathbf{e}_3\cos\theta'. \tag{9}$$

In this paper, we restrict our consideration to beams with

$$\theta' = \kappa_0\theta, \quad \varphi' = \varphi, \tag{10}$$

where κ_0 is some real parameter; $0 < \kappa_0 \leq 1$. Correspondingly, to set the beam base, it is convinient to use the radial, the meridional, and the azimuthal basis vectors

$$\mathbf{e}_r(\theta',\varphi) = \sin\theta'(\mathbf{e}_1\cos\varphi + \mathbf{e}_2\sin\varphi) + \mathbf{e}_3\cos\theta', \tag{11}$$

$$\mathbf{e}_{\theta'}(\theta',\varphi) = \cos\theta'(\mathbf{e}_1\cos\varphi + \mathbf{e}_2\sin\varphi) - \mathbf{e}_3\sin\theta', \tag{12}$$

$$\mathbf{e}_\varphi(\varphi) = -\mathbf{e}_1\sin\varphi + \mathbf{e}_2\cos\varphi. \tag{13}$$

Let us set two amplitude functions by

$$\mathbf{W}(\theta,\varphi) \equiv \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \mathbf{e}_{\theta'} \\ \mathbf{e}_\varphi \end{pmatrix}, \tag{14}$$

$$\mathbf{W}(\theta,\varphi) \equiv \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \mathbf{e}_\varphi \\ -\mathbf{e}_{\theta'} \end{pmatrix}. \tag{15}$$

Since the beams with the amplitude function $\mathbf{W}$ [Eq. (14)] are composed from plane waves with the meridional orientation of $\mathbf{E}$ and the azimuthal orientation of $\mathbf{B}$, they will be referred to as E_M beams or B_A beams. Similarly, the amplitude function $\mathbf{W}$ [Eq. (15)] results in E_A beams or B_M beams. The field vectors of E_M and E_A beams are related by the duality transformation $\mathbf{E} \to \mathbf{B},\ \mathbf{B} \to -\mathbf{E}$.

3. Orthonormal Beams

Let us first consider orthonormal beams with the angular spectrum $\Omega = 2\pi$, i.e., the superpositions of eigenwaves propagating into a given halfspace. To this end, let us set $\theta_1 = 0$ and $\theta_2 = \pi/2$ in Eq. (1), and $\kappa_0 = 1$ in Eq. (10). In this case, the amplitude functions $\mathbf{W}(\theta,\varphi)$ for E_M and E_A beams are given by Eqs. (14) and (15) with $\theta' = \theta$, and the orthonormalizing function $\nu = \nu(\theta,\varphi)$ reduces to a constant [5]. The beam manifold $\mathcal{B} = S^2_N$ is the northern hemisphere S^2_N of the unit sphere S^2. This results in two different sets of orthonormal beams defined by the spherical harmonics Y^s_j with even and odd j, respectively. However, it is of value to have a complete system of orthonormal beams $\mathbf{W}^s_j$ defined by the whole set of spherical harmonics Y^s_j, for which $\langle \mathbf{W}^s_j|Q|\mathbf{W}^{s'}_{j'}\rangle = 0$, if at least one of the three conditions is met: $j'\neq j$, $s'\neq s$, or the beams have the alternative polarization states (E_M and E_A beams).

To this end, let us set the beam base of time–harmonic electromagnetic E_M beams $\mathbf{W}^s_j$ [Eq. (1)] by Eqs. (9) and (10)–(15) with $\theta_1 = 0$, $\theta_2 = \pi$, and $\kappa_0 \leq 1/2$. In this case, the beam manifold is the unit sphere ($\mathcal{B} = S^2$), the angular spectrum $\Omega \leq 2\pi$, and the orthonormalizing function has the form

$$\nu(\theta) = \frac{2}{\lambda}\sqrt{\frac{2\pi\kappa_0 N_Q \sin(\kappa_0\theta)}{c\sin\theta}}. \tag{16}$$

These E_M beams also can be expanded into a series as described in Ref. [5]. As before, E_M and E_A beams are related by the duality transformation.

The smaller is κ_0, the smaller is the angular spectrum Ω, i.e., the more collimated is a beam. Conversely, if $\kappa_0 = 1/2$, i.e., $\Omega = 2\pi$, the beam becomes highly localized and has an energy distribution in the core region similar to the beams presented in Refs. [5, 6, 7]. When $s\neq 0$ and $\kappa = 1/2$, or $\kappa \approx 1/2$, these beams resemble electromagnetic tornadoes with spiral energy fluxes and pronounced core regions.

The general time–harmonic beam with two–dimensional beam manifold $\mathcal{B}$ can be written as

$$\mathbf{W}(\mathbf{r},t) = e^{-i\omega t}\int_{\mathcal{B}} e^{i\mathbf{r}\cdot\mathbf{k}(b)}\nu(b)u(b)\mathbf{W}(b)d\mathcal{B}, \tag{17}$$

where $u : \mathcal{B} \to C^1$ is a complex scalar function on $\mathcal{B}$. Let (u_n) be an orthonormal base of complex functions on $\mathcal{B}$. Then, the function u can be expanded into a series as

$$u(b) = \sum_n c_n u_n(b), \tag{18}$$

where $c_n = \langle u_m|u\rangle$. By applying the approach described in Refs. [5, 8], we obtain an expansion of $\mathbf{W}$ (17) into a series of orthonormal beams $\mathbf{W}_n$ as

$$\mathbf{W} = \sum_n c_n \mathbf{W}_n. \tag{19}$$

It is essential that the coefficients c_n can be extracted from the beam $\mathbf{W}$ as follows:

$$c_n = \frac{1}{N_Q}\langle \mathbf{W}_n|Q|\mathbf{W}\rangle. \tag{20}$$

What is even more important they are measurable values provided that there exists a source of orthonormal beams $\mathbf{W}_n$. As it is shown in Refs. [5, 7], $\mathcal{I} = \langle \mathbf{W}|Q|\mathbf{W}\rangle$ is the energy flux through the plane σ_0 in the case of time–harmonic beams with two–dimensional manifold $\mathcal{B}$. Each of the complex coefficients c_n of the beam $\mathbf{W}$ (19) can be calculated from the results of three measurements [5, 7].

4. Conclusion

The presented approach to characterizing complex media is based on the use of new families of orthonormal electromagnetic beams. Each familly consists of orthonormal exact solutions of Maxwell's equations, which differ fundamentally from the well–known approximate solutions —the Hermite–Gaussian and Laguerre–Gaussian beams. By using these solutions, the results obtained in Refs. [1, 2, 3, 4] for the case of plane incident waves are generalized to the case of time–harmonic beams obliquely incident onto a general bianisotropic slab. To this end, the fields of incident, reflected, and transmitted waves are expanded into series of orthonormal vector functions. The obtained solutions make it possible to calculate the complex scalar coefficients of these series. It is shown that these coefficients are measurable values, and the corresponding measurement scheme is suggested. Assuming that they are given or measured, it is possible to reconstruct the reflection and transmission coefficients of the slab for partial incident plane waves and then, using the techniques presented in Refs. [1, 2, 3, 4], to extract the whole set of material parameters. One can use various families of orthonormal beams, in particular, the family of beams defined by the spherical harmonics. Results of numerical analysis of the latter and peculiarities of its possible application to characterizing various complex media will be reported orally.

The proposed approach makes it possible to use beams with wide angular spectrum, focused into a small spot on the sample surface. Usage of well focused beams eliminates the need to work in an anechoic enviroment.

References

[1] G. N. Borzdov, "Free space measurement techniques for characterizing anisotropic, chiral and bianisotropic media," in *Proc. Bianisotropics'98*, Braunschweig, Germany, June 1998, pp. 261–264.

[2] G. N. Borzdov, "An optimization of free space measurement schemes for characterizing complex media," in *Proc. Bianisotropics'98*, Braunschweig, Germany, June 1998, pp. 301–304.

[3] G. N. Borzdov, "On the measurement of material parameters of a general bianisotropic medium," in *Proc. PIERS'98*, Nantes, France, July 1998, p. 516.

[4] G. N. Borzdov, "Inverse problem of reflection and transmission for a bianisotropic medium," in *Advances in Complex Electromagnetic Materials* (A. Priou *et al.*, eds). Dordrecht: Kluwer, 1997, pp. 71–84.

[5] G. N. Borzdov, "Plane–wave superpositions defined by orthonormal scalar functions on two– and three–dimensional manifolds," *Phys. Rev. E*, vol.61, no. 4, pp. 4462–4478, April 2000.

[6] G. N. Borzdov, "New types of electromagnetic beams in complex media and free space," in *Abstracts of Millennium Conference on Antennas & Propagation AP2000*, Davos, Switzerland, April 2000, Vol. II – Propagation, p. 228.

[7] G. N. Borzdov, "Electromagnetic beams defined by the spherical harmonics with applications to characterizing complex media," in *Abstracts of Millennium Conference on Antennas & Propagation AP2000*, Davos, Switzerland, April 2000, Vol. II – Propagation, p. 229.

[8] G. N. Borzdov, "New types of orthonormal electromagnetic beams in complex media and free space", this issue.