Diffraction by a Conducting Half-Plane in a Chiroplasma

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Abstract

The scattering of a plane eigenwave normally incident on a half-plane placed in a chiroplasma whose distinguished axis is parallel to the edge of the screen is considered. The formulation of the problem leads to the vector functional equation which is exactly solved by the Wiener–Hopf–Hilbert method. Some distinct properties of the diffraction problem are noted.

1. Introduction

Diffraction of waves as an item of bianisotropy research in electromagnetics is in need of a unimaginative but adequate model description. As for chiral media subjected to an external magnetic field, the constitutive relations of a chiroplasma [1] are rather simple. They differ favorably from a previous ones [2] by what properly furnish the transition to an isotropic chiral medium. Therefore we use the chiroplasma model in the attempt to obtain an exact analytical solution of a half-plane canonical diffraction problem with reference to complex birefringent medium. As opposed to its nonchiral [3], nongyrotropic chiral [4] and biisotropic [5] counterparts, the problem is led to the vector Wiener–Hopf equation. The matrix factorization is fulfilled by virtue of the Wiener–Hopf–Hilbert method. Some features of the obtained exact closed-form solution are discussed then.

2. Statement of the Diffraction Problem

A perfectly conducting screen $x \geq 0$, $y = 0$ is embedded in a chiroplasma whose distinguished z-axis is parallel to the edge. The medium is described by the constitutive relations (time-harmonic factor $\exp(-i\omega t)$ is meant)

$$\left\{ \begin{array}{lcl} \mathbf{D} & = & \overline{\overline{\varepsilon}} \cdot \mathbf{E} + i\xi \mathbf{B} \\ \\ \mathbf{H} & = & i\xi \mathbf{E} + \mathbf{B}/\mu \; . \end{array} \right. \tag{1}$$

The permittivity $\overline{\overline{\varepsilon}}$ depends on the reduced frequencies Ω and R, see explicit definition in [6].

One of two plane eigenwaves with the wave numbers $\varkappa_{1,2}$ where

$$\varkappa_{1,2}^2 = \frac{k_\infty^2}{2} \left[\varepsilon_\perp + \varepsilon_z + a \pm \sqrt{(\varepsilon_\perp + \varepsilon_z - a)^2 + 4a\varepsilon_z} \right], \quad a = 4\mu\varepsilon_\infty^{-1}\xi^2, \quad k_\infty = \omega\sqrt{\varepsilon_\infty \mu} \tag{2}$$

propagates in the sagittal plane $z = 0$ and falls on the half-plane under an angle θ. In this case, the electromagnetic field may be described via the sum of two scalar functions $\varphi_j(x,y)$ $(j = 1,2)$ representing the eigen polarizations with the wave numbers according to Eq. (2). The total field must satisfy the boundary conditions on the screen, where $\hat{\mathbf{e}}_y \times \mathbf{E}^{(t)}(x,0) = 0$, and on its sequel, where $\hat{\mathbf{e}}_y \times \mathbf{E}^{(t)}$ and $\hat{\mathbf{e}}_y \times \mathbf{H}^{(t)}$ are continuous. The scattered field is subjected to the edge condition [7] and the radiation condition at infinity.

The secondary field is sought in the form of the Fourier integral

$$\varphi(x,y) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \Phi(\alpha,y) e^{i\alpha x} d\alpha \tag{3}$$

where $\Phi(\alpha,y)$ yields to the equation

$$\left[\left(\frac{d^2}{dy^2} + \gamma_1^2\right)\right]\left[\left(\frac{d^2}{dy^2} + \gamma_2^2\right)\right]\Phi(\alpha,y) = 0 \tag{4}$$

with non-coincident $\gamma_j = \sqrt{\alpha^2 - \varkappa_j^2}$, $j = 1,2$. The solution of Eq. (4) is

$$\Phi(\alpha,y) = \begin{cases} A_1(\alpha)e^{-\gamma_1 y} + B_1(\alpha)e^{-\gamma_2 y}, & \text{if } y \geq 0 \\ A_2(\alpha)e^{\gamma_1 y} + B_2(\alpha)e^{\gamma_2 y}, & \text{if } y \leq 0 \end{cases} \tag{5}$$

where $A_j(\alpha)$, $B_j(\alpha)$ are amplitude functions to be found. Using Eqs. (3)–(5) we translate the boundary conditions into the spectral domain and present in the matrix form

$$\overline{\overline{\boldsymbol{Q}}}(\alpha) \cdot \boldsymbol{L}(\alpha) = \boldsymbol{E}(\alpha) + \boldsymbol{U}(\alpha) \tag{6}$$

where the two-element column-vectors $\boldsymbol{U}(\alpha)$ and $\boldsymbol{L}(\alpha)$ represent the functions of α which are regular in the regions Π_U, Π_L of the complex α-plane, respectively. The symbols Π_U (Π_L) indicate the upper (lower) half-plane of α including the common regularity strip along the $\mathrm{Re}\,\alpha$-axis. The elements of $\boldsymbol{U}(\alpha)$, $\boldsymbol{L}(\alpha)$ include the unknown amplitude functions whereas the vector $\boldsymbol{E}(\alpha)$ consists of elements relating to components of the transformed electric field of the incident plane wave $\mathbf{E}^{(i)}(x,y)$. Eq. (6) is the vector Wiener–Hopf equation with the matrix kernel $\overline{\overline{\boldsymbol{Q}}}(\alpha) \equiv (Q_{ij}(\alpha))$ where

$$Q_{11}(\alpha) = \frac{k_\infty^2 \varepsilon_\perp - \varkappa_1^2}{\gamma_1} - \frac{k_\infty^2 \varepsilon_\perp - \varkappa_2^2}{\gamma_2}, \quad Q_{12}(\alpha) = \frac{g\alpha}{\varepsilon\gamma_2}(\varkappa_1^2 - \varkappa_2^2)$$
$$Q_{21}(\alpha) = \frac{g}{\varepsilon}\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right), \quad Q_{22}(\alpha) = \gamma_1 - \gamma_2 \frac{k_\infty^2 \varepsilon_\perp - \varkappa_1^2}{k_\infty^2 \varepsilon_\perp - \varkappa_2^2} - \frac{g^2\alpha^2}{\varepsilon^2\gamma_2} \frac{\varkappa_1^2 - \varkappa_2^2}{k_\infty^2 \varepsilon_\perp - \varkappa_2^2}.$$

3. Solution of the Functional Vector Equations

In order to perform the fundamental step in the Wiener–Hopf technique, that is to decompose the matrix $\overline{\overline{\boldsymbol{Q}}}(\alpha)$ in the form of a product $\overline{\overline{\boldsymbol{Q}}}(\alpha) = \overline{\overline{\boldsymbol{Q}}}_U(\alpha) \cdot \overline{\overline{\boldsymbol{Q}}}_L(\alpha)$ we use the Hurd idea [8] and re-formulate the homogeneous version of Eq. (6) as a vector Hilbert problem on the branch cuts $\Gamma_{1,2}$ due to the branch points $\alpha = \varkappa_{1,2}$ in Π_U. Let the "+" and "−" subscripts indicate the values of functions at the opposite sides of $\Gamma_{1,2}$. After elimination of $\boldsymbol{U}(\alpha)$, one obtains a vector Hilbert problem

$$\boldsymbol{L}_+(\alpha) = \overline{\overline{\boldsymbol{H}}}(\alpha) \cdot \boldsymbol{L}_-(\alpha), \quad \text{where } \overline{\overline{\boldsymbol{H}}}(\alpha) = \overline{\overline{\boldsymbol{Q}}}_+^{-1}(\alpha) \cdot \overline{\overline{\boldsymbol{Q}}}_-(\alpha)\,. \tag{7}$$

The matrix $\overline{\overline{\boldsymbol{H}}}(\alpha)$ has zero trace and contains only polynomial elements. We have for the contours Γ_1 and Γ_2

$$\overline{\overline{\boldsymbol{H}}}(\alpha)_{|\Gamma_1} = -\overline{\overline{\boldsymbol{H}}}(\alpha)_{|\Gamma_2} = \frac{1}{\Delta_+}\begin{pmatrix} l & m \\ n & -l \end{pmatrix} \tag{8}$$

where

$$\begin{cases} l &= \gamma_2^2(k_\infty^2\varepsilon_\perp - \varkappa_1^2)^2 - \gamma_1^2(k_\infty^2\varepsilon_\perp - \varkappa_2^2)^2 - \frac{g^2}{\varepsilon^2}\alpha^2(\varkappa_1^2 - \varkappa_2^2)^2 \\ m &= 2\frac{g}{\varepsilon}\alpha\gamma_1^2(\varkappa_1^2 - \varkappa_2^2)(k_\infty^2\varepsilon_\perp - \varkappa_2^2) \\ n &= -2\frac{g}{\varepsilon}\alpha(\varkappa_1^2 - \varkappa_2^2)(k_\infty^2\varepsilon_\perp - \varkappa_2^2) \end{cases} \tag{9}$$

and

$$\Delta(\alpha) = \frac{g^2}{\varepsilon^2}\alpha^2(\varkappa_1^2 - \varkappa_2^2)^2 - [\gamma_1(k_\infty^2\varepsilon_\perp - \varkappa_2^2) - \gamma_2(k_\infty^2\varepsilon_\perp - \varkappa_1^2)]^2. \tag{10}$$

According to the Wiener–Hopf–Hilbert method, we introduce another unknown vector $\boldsymbol{V}(\alpha) = \overline{\overline{\boldsymbol{T}}}(\alpha) \cdot \boldsymbol{L}(\alpha)$ in order to receive a new vector Hilbert problem for $\boldsymbol{V}(\alpha)$. Due to special choice of $\overline{\overline{\boldsymbol{T}}}(\alpha)$, the latter may be separated into two uncoupled standard Hilbert problems [9], whereupon the vectors $\boldsymbol{V}(\alpha)$ and $\boldsymbol{L}(\alpha)$ are found. These vectors should be regular in the domain $\Pi_U + \Pi_L - \Gamma_1 - \Gamma_2$. Farther, one may construct two vectors $\boldsymbol{L}^{(1,2)}(\alpha) \equiv (L_i^{(1,2)}(\alpha))$ which fulfil the condition $L_1^{(1)}L_2^{(2)} - L_1^{(2)}L_2^{(1)} \neq 0$. We use them in order to form the matrix $\overline{\overline{\boldsymbol{Q}}}_L^{-1}(\alpha)$ which also should satisfy Eq. (7). The matrices $\tilde{\overline{\overline{\boldsymbol{Q}}}}_L(\alpha)$ and $\tilde{\overline{\overline{\boldsymbol{Q}}}}_U^{-1}(\alpha) = \tilde{\overline{\overline{\boldsymbol{Q}}}}_L(\alpha) \cdot \tilde{\overline{\overline{\boldsymbol{Q}}}}^{-1}(\alpha)$ have not branch points in Π_L and Π_U, respectively, but temporarily have poles there. The next step is to cancel the undesirable singularities in these matrices using a properly chosen rational matrix $\overline{\overline{\boldsymbol{F}}}(\alpha)$, namely

$$\overline{\overline{\boldsymbol{Q}}}_U^{-1}(\alpha) = \overline{\overline{\boldsymbol{F}}}(\alpha) \cdot \tilde{\overline{\overline{\boldsymbol{Q}}}}_U^{-1}(\alpha), \quad \overline{\overline{\boldsymbol{Q}}}_L(\alpha) = \overline{\overline{\boldsymbol{F}}}(\alpha) \cdot \tilde{\overline{\overline{\boldsymbol{Q}}}}_L(\alpha)\,. \tag{11}$$

To define the elements of $\overline{\overline{\boldsymbol{F}}}(\alpha)$ we have to take into account the number of roots of $\Delta(\alpha)$ located on its branch, which is given via the conditions at infinity. It turns out, that at least four constants can be chosen arbitrary. The correct definition allows to simplify essentially the further complicated calculations.

4. Completion and Analysis of the Solution

When the matrix factors $\overline{\overline{\boldsymbol{Q}}}_{U,L}(\alpha)$ are found the solution of Eq. (6) is straightforward. It is rearranged so that to receive the left- and right-hand sides which are regular in Π_U and Π_L, respectively, and define an integral function $J(\alpha)$. According to the edge condition, the asymptotic behaviour of both sides of the equation at $|\alpha| \to \infty$ permits to put $J(\alpha) = 0$. This makes it possible to find the spectral amplitudes $A_j(\alpha)$, $B_j(\alpha)$ and to complete the solution of the problem.

It turns out that the integrand of Eq. (3) is quite cumbersome. Yet the related integrals are suitable for the asymptotic evaluation of far fields by the method of steepest descent. The singularities of the integrands give rise to the distinct wave species. The pole contributions are coupled with the geometrical optics field which in the illuminated region $y \geq 0$ consists of the reflected basic mode and the concominant mode excited due to reflection coupling between modes. An additional pole originated from the dispersion relation $\Delta(\alpha) = 0$ (see Eq. (10)) contributes a surface-wave term. The unidirectional surface wave [6] propagates along a face of the half-plane. Reversion of the external magnetic field shifts it on the other face. The saddle-point contributions are interpreted as two congruences of diffracted rays. At last, in principle, one needs to consider the branch-point contributions that lead to the such manifestation of modal coupling and total reflection as lateral waves.

5. Conclusion

A problem of plane wave diffraction by a perfectly conducting screen in a homogeneous chiro-plasma is solved for the case of normal incidence on the edge which is parallel to the external magnetic field. The formulation leads to the vector Wiener–Hopf equation which is solved by the Wiener–Hopf–Hilbert method. Because of numerous wave species involved, the problem has certain versatility. An outline is conducted on the example of both propagating bulk eigenwaves of the medium. A unidirectional surface wave and lateral waves are the most notable features of the far field in the diffraction problem under consideration.

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