Singular Guided Waves in a Chiroplasma

V. V. Fisanov[1] and D. A. Marakassov[2]

[1] Siberian Physical and Technical Institute, Tomsk State University
Revolution Square 1, 634050 Tomsk, Russia
Fax: +7–3822–233–034; email: kanc@spti.tsu.ru

[2] Department of Radiophysics, Tomsk State University
Lenin Avenue 36, 634050 Tomsk, Russia;
Electromagnetics Laboratory, Helsinki University of Technology
P.O. Box 3000, FIN-02015 HUT, Finland

Abstract

Waves with linearly distance-dependent amplitude (singular waves) directed by one and two plane boundaries in a chiroplasma are investigated for the Voigt geometrical configuration. Specially selected values of the chirality parameter give a transparent and an opaque version of the medium with twofold wave numbers for bulk eigenwaves. Dispersion relations are derived and solved with respect to electric or magnetic walls and an interface of two media.

1. Introduction

Waves with distance-dependent amplitude (so-called *singular waves*) correspond to multiple roots of the related secular equation. They are known in optics of absorbing and transparent crystals (see e.g. [1], [2]). Such a form of solution is suitable especially well for waves guided by different boundaries. In particular, singular surface polaritons and magnetoplasmons may propagate along the plane surface of an anisotropic crystal [3] and a magnetoplasma interface in the Faraday geometric configuration [4]. We consider singular guided waves in a chiroplasma in the case of the Voigt configuration, when these waves do not exist in a nonchiral magnetoplasma [5].

2. Transverse Propagating Singular Eigenwaves

A chiroplasma is the gyroelectric version of the Faraday chiral media described by the well-known constitutive relations [6]

$$\left\{ \begin{array}{rcl} \mathbf{D} &=& \overline{\overline{\varepsilon}} \cdot \mathbf{E} + i\xi \mathbf{B} \\ \\ \mathbf{H} &=& i\xi \mathbf{E} + \mathbf{B}/\mu \end{array} \right. \tag{1}$$

where

$$\overline{\overline{\varepsilon}} = \varepsilon_\infty \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \tag{2}$$

is the permittivity tensor for the biasing magnetostatic field directed along the z-axis. Its elements depend on reduced frequency parameters Ω and R as usual [7]. The time-harmonic $\exp(-i\omega t)$ dependence is meant.

We study transverse to the z-axis eigenwave propagation. In this case, one can introduce a scalar function $\varphi(x,y)$ satisfying the fourth-order wave equation

$$\left\{\Delta_\perp^2 + \left[k_\infty^2\left(\varepsilon_\perp + \varepsilon_z\right) + 4\omega^2\mu^2\xi^2\right]\Delta_\perp + k_\infty^4\varepsilon_\perp\varepsilon_z\right\}\varphi\left(x,y\right) = 0 \tag{3}$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $k_\infty = \omega\sqrt{\varepsilon_\infty\mu}$, and $\varepsilon_\perp = \varepsilon - g^2/\varepsilon$ is the Voigt permittivity. The differential operator allows to fulfil the factorisation procedure

$$\left[\left(\Delta_\perp + k_\infty^2\kappa_+^2\right)\left(\Delta_\perp + k_\infty^2\kappa_-^2\right)\right]\varphi\left(x,y\right) = 0 \tag{4}$$

where the eigenvalues

$$\kappa_\pm^2 = \frac{1}{2}\left[\varepsilon_\perp + \varepsilon_z + 4a^2 \pm \sqrt{\left(\varepsilon_\perp - \varepsilon_z + 4a^2\right)^2 + 16a^2\varepsilon_z}\right] \tag{5}$$

define two possible kinds of field polarization, $a = \xi\sqrt{\mu/\varepsilon_\infty}$ is the normalized chirality admittance. Unlike an isotropic chiral medium, the wave numbers κ_+ and κ_- can be equal here under condition that the square root in Eq. (5) is zero. This gives critical values of $a = \pm a_{1,2}$ where

$$a_{1,2} = \tfrac{1}{2}\left(\sqrt{-\varepsilon_z} \pm \sqrt{-\varepsilon_\perp}\right) \tag{6}$$

if both $\varepsilon_\perp$ and ε_z are negative. It is valid inside the frequency range

$$0 < \Omega^2 < \frac{1}{2}\left[R^2 + 2 - \sqrt{R^2(R^2+4)}\right]. \tag{7}$$

Both values of a (6) make possible to consider a chiroplasma as a nonchiral gyroelectric unirefringent medium characterized by the wave equation

$$\left[\left(\Delta_\perp + k_\infty^2\kappa^2\right)^2\right]\varphi\left(x,y\right) = 0 \tag{8}$$

with the twofold wave number $\kappa = \kappa_1 = \sqrt[4]{\varepsilon_z\,\varepsilon_\perp}$ (this is the geometric mean between the indices of the ordinary and extraordinary waves in a nonchiral magnetoplasma) or $\kappa = \kappa_2 = i\sqrt[4]{\varepsilon_z\,\varepsilon_\perp}$.

The general form of solution of Eq. (8) differs from the usual homogeneous plane-wave representation in respect of the amplitude factor which is now distance-dependent. Hereafter, we intend to concentrate on waves guided along the x-axis (it is referred to as the Voigt geometric configuration), therefore we prefer to write down the partial solution of Eq. (8) as follows:

$$\varphi\left(x,y\right) = \left(Z_1 + Z_2 y\right)\exp\left[ik_\infty\left(\alpha x + \beta y\right)\right], \qquad \alpha^2 + \beta^2 = \kappa^2 \tag{9}$$

where Z_1 and Z_2 do not depend on the coordinates. Expressions for the singular-wave components of $\mathbf{E}$ and $\mathbf{H}$ are derived from the Maxwell and Helmholtz equations in conjunction with Eq. (1).

3. Singular Surface Waves Guided by a Boundary or an Interface

Let the half-space of the medium $y > 0$ be bounded by a plane screen or an interface of infinite extent. The boundary is able to trap a singular surface wave having real α and pure imaginary β because the exponential attenuation exceeds the linear growth in the y direction.

An electric wall. The boundary condition $\mathbf{e}_y \times \mathbf{E}\left(x,0\right) = 0$ leads to the set of linear algebraic equations

$$\begin{cases} Z_2 + k_\infty\left(\frac{g}{\varepsilon}\alpha + i\beta\right)Z_1 & = 0 \\ 2i\beta Z_2 + k_\infty\left(\varepsilon_\perp - \kappa^2\right)Z_1 & = 0. \end{cases} \tag{10}$$

To find nontrivial solutions of Eq. (10) its determinant should be set equal to zero. This allows to obtain the secular equation

$$\varepsilon_\perp - \kappa^2 - 2i\beta\left(\frac{g}{\varepsilon}\alpha + i\beta\right) = 0\,. \tag{11}$$

This equation is solved exactly. A unidirectional surface wave exists only if $\varepsilon > 0$ and it propagates with the coefficient $\alpha = |\alpha| \operatorname{sgn} g$ where

$$\begin{pmatrix} |\alpha| \\ |\beta| \end{pmatrix} = \frac{1}{2}\left[\left(\sqrt{\varepsilon - \varepsilon_z} + \sqrt{\varepsilon - \varepsilon_\perp}\right)^2 - \left(\sqrt{-\varepsilon_z} \mp \sqrt{-\varepsilon_\perp}\right)^2\right]^{1/2}. \tag{12}$$

Eq. (12) is valid for the case of the transparent medium ($\kappa = \kappa_1$). Option $\kappa = \kappa_2$ (the case of the opaque medium) needs to interchange $|\alpha|$ and the decrement $|\beta|$.

It is worthwhile to compare this hybrid singular wave with the unidirectional TEM surface wave on a perfectly conducting electric screen placed in a magnetoplasma [5]. That wave has $|\alpha| = \sqrt{\varepsilon}$, its low-frequency branch exists for $\Omega < R$ where bulk waves do not propagate ($\kappa = \sqrt{\varepsilon_\perp}$ is purely imaginary).

A magnetic wall. Applying the boundary condition $\mathbf{e}_y \times \mathbf{H}(x,0) = 0$ we receive the dispersion equation

$$2\left(\sqrt{-\varepsilon_z} \pm 3\sqrt{-\varepsilon_\perp}\right)\beta^2 \mp \sqrt{-\varepsilon_\perp}\left(\varepsilon_\perp - \varepsilon_z\right) - 2i\frac{g}{\varepsilon}\frac{\left(\sqrt{-\varepsilon_z} \pm \sqrt{-\varepsilon_\perp}\right)^2}{\sqrt{-\varepsilon_z} \mp \sqrt{-\varepsilon_\perp}}\alpha\beta = 0 \tag{13}$$

which describes also a unidirectional singular surface wave. Here the upper sign corresponds to $\kappa = \kappa_1$ and the lower one to $\kappa = \kappa_2$. After a hyperbolic substitution, Eq. (13) is transformed to a quadratic equation. For $\kappa = \kappa_1$, the direction of propagation is defined according to $\operatorname{sgn}(g\varepsilon\alpha) = 1$, and the wave exists inside the band (7). For $\kappa = \kappa_2$, there is a gap in the dispersion diagram if R is small.

An interface of two enantiomorphous media. In order to find the dispersion relation in the case of an interface of two mirror-conjugate media

$$\left(1 \pm \frac{\sqrt{\varepsilon_\perp \varepsilon_z}}{\varepsilon}\right)\left(\sqrt{-\varepsilon_z} \pm \sqrt{-\varepsilon_\perp}\right)^2\beta^2 + \frac{1}{4}\left(\varepsilon_\perp - \varepsilon_z\right)^2 - \left(4 + \frac{\left(\sqrt{-\varepsilon_z} \pm \sqrt{-\varepsilon_\perp}\right)^2}{\varepsilon}\right)\beta^4 = 0 \tag{14}$$

(where the upper and lower signs correspond to the transparent and nontransparent media, respectively) one has to require continuity of tangential electric and magnetic fields at the interface $y = 0$. Singular surface wave propagates symmetrically in both directions within the band (7). In the opaque medium the pass band has the lower cut off bound if $R^2 < 4(4 + 3\sqrt{2})$.

4. Singular Waves of a Parallel–Plate Waveguide

Let us consider two parallel electric or magnetic walls $y = 0$ and $y = d$ bounding the waveguide region entirely filled by chiroplasma. In order to describe the singular field one needs to take a superposition of two countersolutions in the form of Eq. (9). Transcendental dispersion relations

$$4\frac{\varepsilon_\perp}{\varepsilon\left(\varepsilon_\perp - \kappa^2\right)^2}\beta^2\left(\beta^2 + \varepsilon - \kappa^2\right) + 1 - \left[\frac{k_\infty \beta d}{\sin\left(k_\infty \beta d\right)}\right]^2 = 0 \tag{15}$$

(for electric walls) and

$$8\beta^2\frac{2\beta^2\left[\left(\varepsilon_\perp + a^2\right)^2 - a^4\frac{g^2}{\varepsilon^2}\right] - \left(\varepsilon_\perp + a^2\right)\left(\varepsilon_\perp + \kappa^2\right)\left(\varepsilon_\perp + 2a^2 - \kappa^2\right) + 2a^4\frac{g^2}{\varepsilon^2}\kappa^2}{\left(\varepsilon_\perp + 2a^2 - \kappa^2\right)^2\left(\varepsilon_\perp + \kappa^2\right)^2} + 1 - \left[\frac{k_\infty \beta d}{\sin\left(k_\infty \beta d\right)}\right]^2 = 0 \tag{16}$$

(for magnetic walls) contain a common trigonometric part. Dispersion features of singular waves are quite different in the cases with $\kappa = \kappa_1$ and $\kappa = \kappa_2$. For the opaque medium, the waveguide slightly resembles the appropriate perfectly conducting screen, however an additional frequency stop-band appears for the variant with magnetic walls. The behaviour of singular guided waves in the transparent medium is more complicated because their transverse wave numbers may have real values. Under certain conditions, a singular waveguide supports one slow and a few fast waves.

5. Conclusion

The singular surface electromagnetic waves are guided by a plane boundary of a chiroplasma half-space if chirality and plasma parameters are properly matched. They form a complete set of surface polaritons jointly with the Rayleigh surface waves and generalized surface waves [7], [8] in the Voigt geometry. It is shown that a parallel-plate waveguide supports under certain conditions singular propagating modes whose characteristics drastically depend on the type of the boundary conditions.

Acknowledgement
D. Marakassov thanks the Helsinki University of Technology for a study term in the year 2000 and financial support.

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