Characterization of Rectangular Waveguide with a Pseudochiral Ω Slab

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Abstract

Mode matching approach is applied to a rectangular waveguide containing a pseudochiral Ω slab. From waveguide measurements carried over a few placement of the slab in the guide the constitutive parameters are extracted. The extraction is performed numerically by a algorithm minimizing the differences between theoretical and measured values of scattering parameters.

1. Introduction

The study of the pseudochiral Ω guides is important for several reasons. Electromagnetic properties introduced by Ω media can enhance the guide performance [1] or provide some added field phenomena over the guide behaviour [2],[3]. Therefore it is important to predict the effect that the Ω medium has on their propagation and scattering characteristics. This paper investigates the structure of the rectangular waveguide loaded with an Ω slab as shown in Fig. 1. We assume that the Ω slab is extended over the waveguide height and can be arbitrary placed in the guide. The excitation is chosen to be TE_{no} modes of the input rectangular waveguides so that due to the localization of the Ω particles in the slab the scattered fields can be defined only by TE_{no} waves [3]. Taking into account the above restrictions the problem is solved using the mode matching approach. It yields very reliable and accurate values for scattering parameters. The knowledge of this scattering characteristics for a few different localizations of the slab in the rectangular guide is basis for the extraction of the dispersive, complex Ω medium parameters.

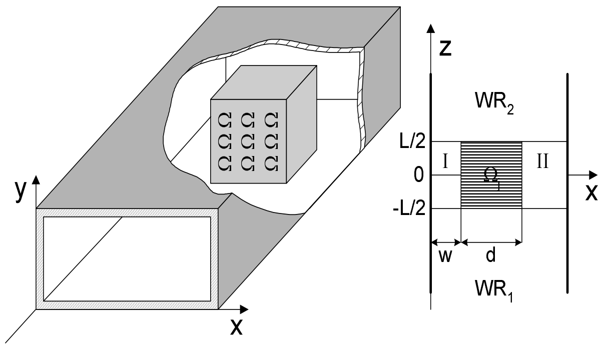

Figure 1: Rectangular waveguide with Ω slab and geometry of the investigated structure.

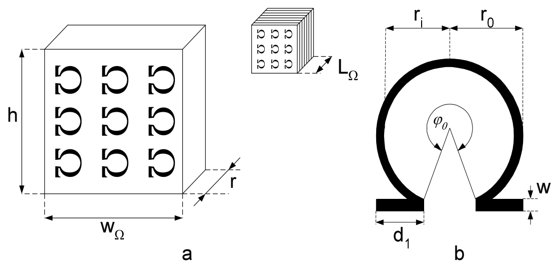

Figure 2: (a) - An Ω sample used in experiment, parameters: $w_\Omega = 10mm$, $l_\Omega = 10mm$, $h = 10.2mm$, $r = 0.2mm$; (b) - A single Ω element, parameters: $r_i = 0.915mm$, $r_o = 1.046mm$, $w = 0.131mm$, $d = 2 * d_1 = 1.6mm$, $\varphi_0 = 5.96rad$

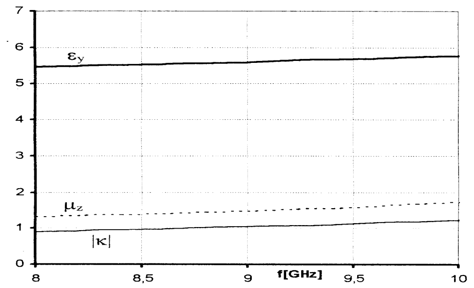

Figure 3: Material parameters computed for static capacity and inductance.

2. Specification of the Slab

Let us consider a three dimensional (3-D) slab of Ω medium as shown in Fig. 1 whose the relative electric permittivity and magnetic permeability tensors are of the diagonal form: $\bar{\varepsilon} = \varepsilon(\vec{i}_x\vec{i}_x + \vec{i}_z\vec{i}_z) + \varepsilon_y\vec{i}_y\vec{i}_y$ and $\bar{\mu_c} = \mu(\vec{i}_x\vec{i}_x + \vec{i}_y\vec{i}_y) + \mu_z\vec{i}_z\vec{i}_z$. For the considered pseudochiral slab the magnetoelectric coupling tensors have the following dyadic representation $\bar{\Omega}_{yz} = \Omega\vec{i}_y\vec{i}_z$ and $\bar{\Omega}_{zy} = \Omega\vec{i}_z\vec{i}_y$, where Ω is a coupling admitance between electric and magnetic field along y and z axis respectively. This Ω sample is composed of unilaterally metallized duroid plates ($\varepsilon = 2.2$) with chemically etched matrix of nine Ω particles as illustrated in Fig. 2a. To obtain the Ω block the plates are stuck together using the polystyren glue ($\varepsilon \sim 2.4$).

To calculate the unknown material parameters we used the static method based on the equivalent $L - C$ circuit from M. Saadoun Dr dissertation [2]. Fig. 3 shows plots of the set of material parameters as function of frequency.These values will be used in next steps of extraction as initial parameters.

3. Transfer Matrices Formulation

The problem is solved by using transfer matrix approach (TMA) presented in [3]. Here the TMA is modified to the cases where pseudochiral materials, such as Ω media, are involved. In this paper we get a following transfer matrix equation that defines the relation between the tangential fields components $\underline{F} = (H_z, E_y)^T$ at the side interfaces $x_i = a$ and $x_{i-1} = 0$.

$$\begin{bmatrix} H_z \\ E_y \end{bmatrix}_a = \underbrace{\begin{vmatrix} T_{11}^{II} & T_{12}^{II} \\ T_{21}^{II} & T_{22}^{II} \end{vmatrix}}_{T^{II}} \underbrace{\begin{vmatrix} T_{11}^{\Omega} & T_{12}^{\Omega} \\ T_{21}^{\Omega} & T_{22}^{\Omega} \end{vmatrix}}_{T^{\Omega}} \underbrace{\begin{vmatrix} T_{11}^{I} & T_{12}^{I} \\ T_{21}^{I} & T_{22}^{I} \end{vmatrix}}_{T^{I}} \begin{bmatrix} H_z \\ E_y \end{bmatrix}_0 \quad (1)$$

where: T^I and T^{II} are tangential matrices in free space sectors (I) and (II); T^Ω is a tangential matrix in Ω slab designed in [3].

4. The Scattering Matrix

The knowledge of the results obtained by TMA gives us a possibility calculation of the scattering parameters for the considered structure. The continuity of the transverse electric and magnetic fields over each aperture cross section (see Fig. 1) is expressed as follows:

$$\begin{aligned} \sum_n (a_{1n}+b_{1n})\vec{e}_n &= \sum_n (A_n + B_n e^{-\gamma_{\Omega n} L})\vec{e}_{\Omega n} \\ \sum_n (a_{1n}-b_{1n})\vec{h}_n &= \sum_n (A_n - B_n e^{-\gamma_{\Omega n} L})\vec{h}_{\Omega n} \end{aligned} \tag{2}$$

$$\begin{aligned} \sum_n (a_{2n}+b_{2n})\vec{e}_n &= \sum_n (A_n e^{-\gamma_{\Omega n} L} + B_n)\vec{e}_{\Omega n} \\ \sum_n (-a_{2n}+b_{2n})\vec{h}_n &= \sum_n (A_n e^{-\gamma_{\Omega n} L} - B_n)\vec{h}_{\Omega n} \end{aligned} \tag{3}$$

where: a_{in} and b_{in}, $i = 1, 2$ are complex coefficients of the incident and reflected wave and $\vec{e}_n$, $\vec{h}_n$ are the transverse electric and magnetic field functions in $WR_{1,2}$. A_n and B_n are the forward and backward wave complex coefficients, $\gamma_{\Omega n}$ is the propagation constant and $\vec{e}_{\Omega n}$, $\vec{h}_{\Omega n}$ are the transverse field functions in the Ω_J section which can be solved by TMA described before.
Equations (2) for z = -L/2 and (3) for z = L/2 are converted into a linear matrix form taking the inner products of both side of these equations with eigenfields of orthonormal set of modes in WR guides. In this approach the Ω_J fields are orthogonalized by eigenfields of WR guides. The resulting matrices, truncated to a total of N modes in each of the region, we manipulate so that the amplitudes of the fields in the Ω_J are eliminated. Finally, the solution is expressed in the scattering matrix formulation as

$$\begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_{11} & \underline{\underline{S}}_{12} \\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22} \end{bmatrix} \begin{bmatrix} \underline{a}_1 \\ \underline{a}_2 \end{bmatrix} \tag{4}$$

If the measured and calculated scattering parameters at fixed frequency for a few positions of pseudochiral slab in the guide are known we can use optimization method to determine the parameters of the Ω medium investigated.

5. Optimization

For extraction of the Ω material parameters we use a procedure based on a modified Levenberg-Marquardt algorithm. In this case the problem is stated as follows:

$$\min_{x\in\mathbf{R}} \frac{1}{2}\sum_{i=1}^{m} f_i(\underline{x})^2 \tag{5}$$

where $\underline{x}$ is a search vector of the material dispersive parameters. The considered medium is characterized by three unknown complex values; permittivity $\varepsilon_y = \varepsilon_y^{'} - j\varepsilon_y^{''}$, permeability $\mu_z = \mu_z^{'} - j\mu_z^{''}$ and coupling coefficient $\kappa = \kappa^{'} - j\kappa^{''}$. So we get:

$$\underline{x} = [\varepsilon_y^{'}, \varepsilon_y^{''}, \mu_z^{'}, \mu_z^{''}, \kappa^{'}, \kappa^{''}]^T \tag{6}$$

Other material parameters ($\varepsilon_x = \varepsilon_z = \varepsilon$ and $\mu_x = \mu_y = \mu$) are taken from duroid parameters used in experiment. The goal of optimization procedure is minimization of (5) with

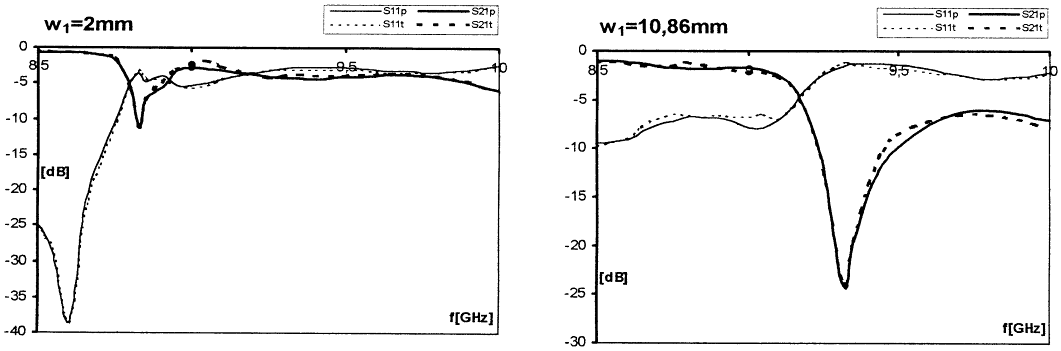

Figure 4: Comparison between optimized and measured performance of the section for a two opposite positions (w) of the Ω slab in the guide.

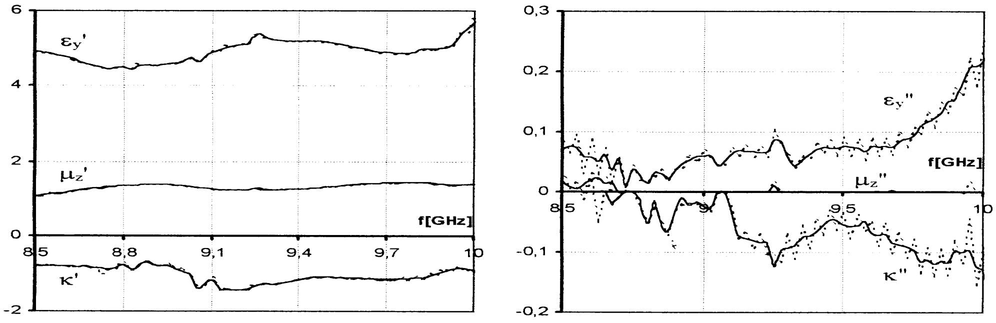

Figure 5: The final material parameters with added noise characteristics.

$f_i(\underline{x}) = |S_{p_i}| - |S_{t_i}|$ where $S_p[dB], S_t[dB]$ - are the measured and calculated scattering parameters, respectively. It is taken as a dominant mode reflection S_{11} and transmission S_{21} coefficients which are measured at fixed frequency and one position of the slab in the guide. Although six goal functions are sufficient for solution of the problem we used twenty six equations in order to correct the slab position errors. One should note from Fig. 4 that after optimization process good agreement between the theoretical and measured characteristics is obtained. In this case the extraction of the Ω material parameters is performed. These results are shown in Fig. 5 as a function of frequency.

6. Conclusion

This paper present the theoretical and measured scattering characteristics of the rectangular waveguide with slab of the Ω medium. These characterisctics are used in the Levenberg-Marquardt optimization algorithm to extract all three constitutive parameters of the Ω material.

References

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