Effective Properties of Multilayered Bianisotropic Systems

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Abstract

A method of introduction of the effective material tensors of bianisotropic multilayered periodic structures based on the approximate calculation of the characteristic matrix of the unit cell of the system with the help of Campbell–Hausdorff series [1] is developed. Obtained effective tensors are valid for the use in a wide wave band. This paper is primarily concerned with a comparison of the accuracy of different approximations, namely, a long wavelength approximation and approximations, using three and five terms of Campbell-Hausdorff series.

1. Introduction

Attention now frequently focuses on the study of the effective properties of plane stratified periodic media. In doing so most often a long wavelength approximation is used. For example, in [2] the coordinate-free formulae for the effective tensors of permittivity ϵ, permeability μ and pseudotensors of gyrotropy α, β of the plane stratified periodic bianisotropic systems of the most general type were obtained. But it is well known [3], [4] that some systems composed from nongyrotropic layers can possess gyrotropic properties due to their specific structure (i.e. possess form gyrotropy). Phenomena of this kind can't be explained in the framework of a long wavelength approximation. To treat such problems it is necessary to extend the wave band, in which the effective tensors can be used. One way of doing this is to employ an approach [5], based on the approximate calculation of the characteristic matrix of the unit cell with the help of Campbell-Hausdorff series. Recently it was shown that by using this approach the effects of form gyrotropy (bianisotropy) can be really explained in terms of the theory of effective parameters (see [6], for example). In [6], [7] our consideration was limited by the three terms of this series.

Here we obtain more accurate formulas using the forth and fifth terms of Campbell-Hausdorff series.

We consider systems formed by a periodic set of plane bianisotropic layers with different thicknesses l_n ($n = 1, 2, \dots N$, where N is the number of the layers forming the unit cell). The layers are characterized by the constitutive relations

$$\mathbf{D}_n = \varepsilon_n \mathbf{E}_n + \alpha_n \mathbf{H}_n,\ \mathbf{B}_n = \beta_n \mathbf{E}_n + \mu_n \mathbf{H}_n, \tag{1}$$

where ε_n, μ_n and α_n, β_n are the dielectric permittivity, the magnetic permeability tensors and the pseudotensors of gyrotropy, respectively.

2. Effective Material Tensor Parameters

The characteristic matrix $P = \exp(ik_0 l M)$ of a layer with thickness l relates the six-vectors $(\mathbf{E}, \mathbf{H})^T$ at the layer boundaries ($k_0 = \omega/c$). M is some matrix depending on the parameters of the layers, angle and plane of incidence. Matrix M is given in an explicit form in [7]. Henceforward we shall follow notation used in [7].

Let us consider the system formed by two alternate layers with different thicknesses l_n and different sets of tensor constants $\varepsilon_n, \mu_n, \alpha_n, \beta_n$ $(n = 1, 2)$. In this instance the characteristic matrix of the unit cell has the form

$$P = \exp(ik_0 LM) = \exp(ik_0 l_2 M_2)\exp(ik_0 l_1 M_1), \tag{2}$$

where $L = l_1 + l_2$ is the system period and M is the matrix to be found. This matrix can be expressed in terms of the layers parameters with the help of Campbell–Hausdorff series as follows

$$M = f_1 M_1 + f_2 M_2 + i f_1 \frac{\pi l_2}{\lambda}[M_2, M_1] - \frac{\pi^2 l_1 l_2}{3\lambda^2}\{f_1[[M_2, M_1], M_1] + f_2[[M_1, M_2], M_2]\} + \cdots \tag{3}$$

where $[M_2, M_1] = M_2 M_1 - M_1 M_2$. If $\pi l_n/\lambda \ll 1$, then the series (3) quickly converges and one can drop the remainder of it after the k–th term. The first and the second terms correspond to the long wavelength limit. The possibilities of introduction of the effective constitutive tensors having regard the third term are discussed in [7]. In what follows we shall break the series after the fifth term.

In perfect analogy with [7] we shall use in (2)–(3) the matrices M_I, M_{1I}, M_{2I}, describing the transformation of the tangential components of the field vectors, instead of M, M_1, M_2, and we assume $\mathbf{q}$ to be the left and the right eigenvector of each of the tensors $\varepsilon_n, \mu_n, \alpha_n, \beta_n$ $(\varepsilon_n \mathbf{q} = \mathbf{q}\varepsilon_n \equiv \varepsilon_{nq}\mathbf{q}, \cdots)$.

To compare the accuracy of different approximations, namely, long wavelength approximation and approximations, using three and five terms of Campbell-Hausdorff series, let us consider the most simple case of normal incidence onto the nongyrotropic nonabsorbing layers of the same thickness $l_1 = l_2 = l$. As this takes place (3) reduces to

$$M_I = \frac{1}{2}(M_{1I} + M_{2I}) + i\frac{\pi l}{2\lambda}\{[M_{2I}, M_{1I}] + i\frac{\pi l}{3\lambda}[[M_{2I}, M_{1I}], M_{1I} - M_{2I}]\} + \cdots \tag{4}$$

Generally speaking, Campbell-Hausdorff series

$$Z = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] + \frac{1}{12}[Y, [Y, X]] + \cdots \tag{5}$$

converges provided $\|X\| < \frac{ln2}{2}, \|Y\| < \frac{ln2}{2}$, where $\|X\| = \left[trace\left(XX^{\dagger}\right)\right]^{\frac{1}{2}}$ is Euclidean valuation of X. In our case series converges if

$$\frac{\pi max(l_1, l_2)}{\lambda} \leq \frac{ln2}{4\ max(\|M_{1I}\|, \|M_{2I}\|)} \tag{6}$$

For example, in the case of nonmagnetic crystals with $\|\varepsilon_I\| \sim 4.5$ (eigenvalues of $\varepsilon_I \sim 3.0$) $\frac{\pi l}{\lambda} \leq 0.039$.

After simple calculation we find the effective material tensors of the system at hand

$$\alpha_I = \beta_I^{\dagger} = i\frac{\pi l}{2\lambda}R_\alpha,\ R_\alpha = (\varepsilon_{1I}\mathbf{q}^{\times}\Delta\mu_I - \Delta\varepsilon_I\mathbf{q}^{\times}\mu_{1I}), \tag{7}$$

$$\varepsilon_I = \frac{1}{2}(\varepsilon_{1I} + \varepsilon_{2I}) + \varepsilon',\ \mu_I = \frac{1}{2}(\mu_{1I} + \mu_{2I}) + \mu', \tag{8}$$

$$\varepsilon' = i\frac{\pi l}{2\lambda}\left(i\frac{\pi l}{3\lambda}\right)R'_\varepsilon,\ R'_\varepsilon = -R_\alpha\mathbf{q}^{\times}\Delta\varepsilon_I - \Delta\varepsilon_I\mathbf{q}^{\times}R_\beta, \tag{9}$$

$$\mu' = i\frac{\pi l}{2\lambda}\left(i\frac{\pi l}{3\lambda}\right) R'_\mu,\ R'_\mu = R_\beta \mathbf{q}^\times \Delta\mu_I + \Delta\mu_I \mathbf{q}^\times R_\alpha, \tag{10}$$

where $\Delta\varepsilon_I = \varepsilon_{2I} - \varepsilon_{1I}$, $\Delta\mu_I = \mu_{2I} - \mu_{1I}$.

It is worth noting that third term of Campbell–Hausdorff series give rise to the effective "tensors of gyrotropy" α_I, β_I while forth and fifth terms contribute only in the effective permittivity and permeability tensors (terms ε', μ' in (8)).

¿From (7),(9), (10) it follows that

$$\|R_\alpha\| \leq 2\left(\|\varepsilon_{1I}\|\ \|\Delta\mu_I\| + \|\Delta\varepsilon_I\|\ \|\mu_{1I}\|\right), \tag{11}$$

$$\|R_\beta\| \leq 2\left(\|\varepsilon_{1I}\|\ \|\Delta\mu_I\| + \|\Delta\varepsilon_I\|\ \|\mu_{1I}\|\right), \tag{12}$$

$$\|R'_\varepsilon\| \leq 2\ \|\Delta\varepsilon_I\|\left(\|R_\alpha\| + \|R_\beta\|\right), \tag{13}$$

$$\|R'_\mu\| \leq 2\ \|\Delta\mu_I\|\left(\|R_\alpha\| + \|R_\beta\|\right). \tag{14}$$

¿From above estimations and formulae (7)-(10) it is clear that in the convergence range the contribution of the forth and fifth terms of Campbell–Hausdorff series in effective tensors is far less than the contribution of third term, and usually may be thought of as negligibly small if compared with the contribution of the first and second terms. But if in the long wavelength approximation the system under hand has transversely isotropic permittivity and permeability tensors, then contribution of the forth and fifth terms can be noticeable. The latter case is of great concern in investigation of form bianisotropy (gyrotropy), because the manifestations of gyrotropy are often suppressed due to the permeability(permittivity) being tensors, especially in the optical wave region [9].

3. Conclusion

In this paper we discussed primarily the comparative contribution of different terms of Campbell–Hausdorff series in the effective tensors of "permittivity, permeability" and "gyrotropy", analyzing the simple case of normal incidence onto the system of nongyrotropic layers. Obtained estimations holds true for gyrotropic layers too, at least for the short enough wavelengths. At normal incidence generalization of formulae (7)–(10) to the case of bianisotropic layers is not a particular problem.

Of course, derived effective tensors are not true constitutive tensors because generally they are valid only in the immediate vicinity of normal incidence. It is possible to introduce true constitutive tensors (i.e. not depending on the angle and plane of incidence) even having regard to five terms in Campbell–Hausdorff series, but only for some systems with the specific relation between the layers parameters. Therefore, proposed method is more convenient for analysis of the effective properties of systems at normal incidence, when there is no restrictions on the parameters of the layers. Extensive computations made with the use of exact and approximate formulas are in good accordance.

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