Blind Estimation of Motion Blur Parameters for Image Deconvolution

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IbPRIA 2007 Girona, June 6-8

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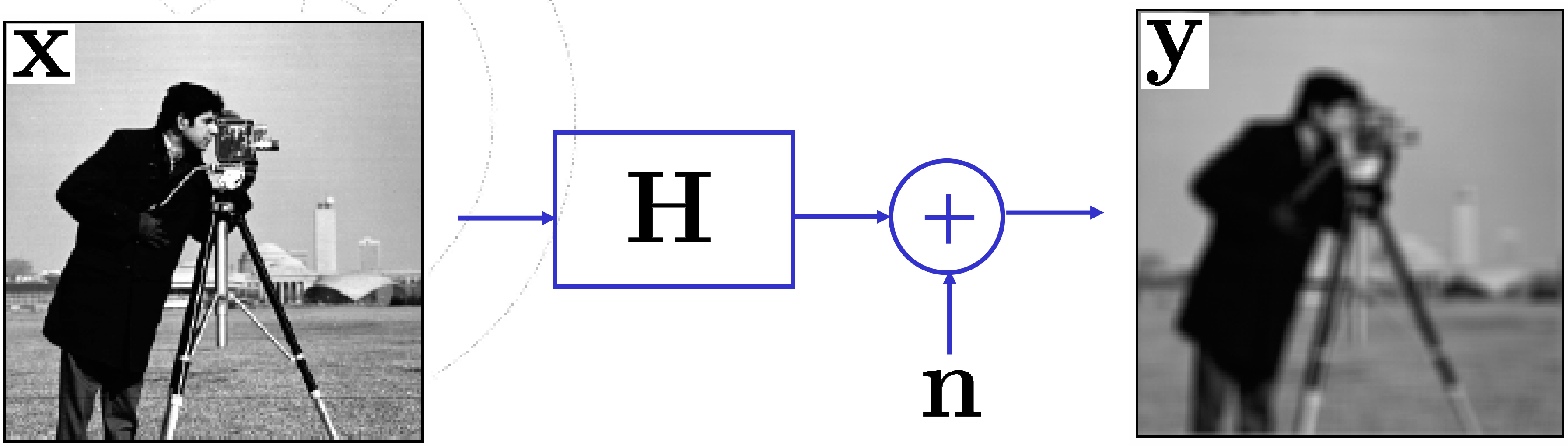
Outline

- Problem formulation
- Motion blur parameters
- Radon transform
- Angle estimation
- Length estimation
- Simulation results
- Conclusions

Problem formulation

Observation model in (linear) image restoration/reconstruction



$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Observed image

Noise

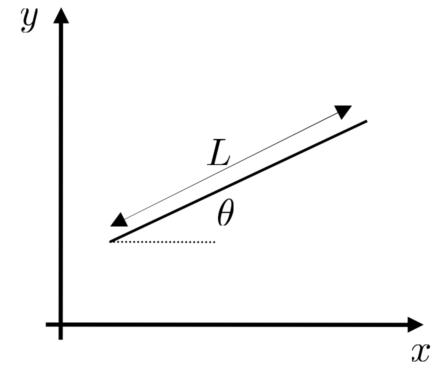
Linear operator
(e.g., blur, tomography,...)

Original image

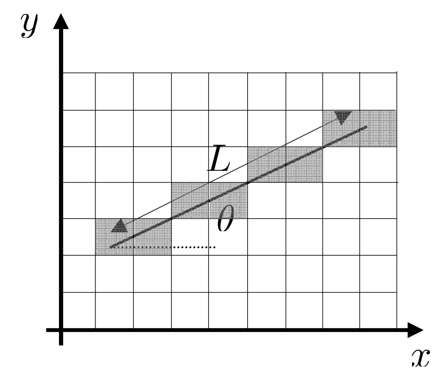
Goal: estimate $\mathbf{H}$ from $\mathbf{y}$

Motion Blur Parameters

- **PSF (continuous formulation):**
  normalized delta function, supported on a line segment with length L and angle θ.



- **PSF (discrete formulation):**
  a straight line segment on a digital grid, obtained by the digital differential analyser (DDA) algorithm.

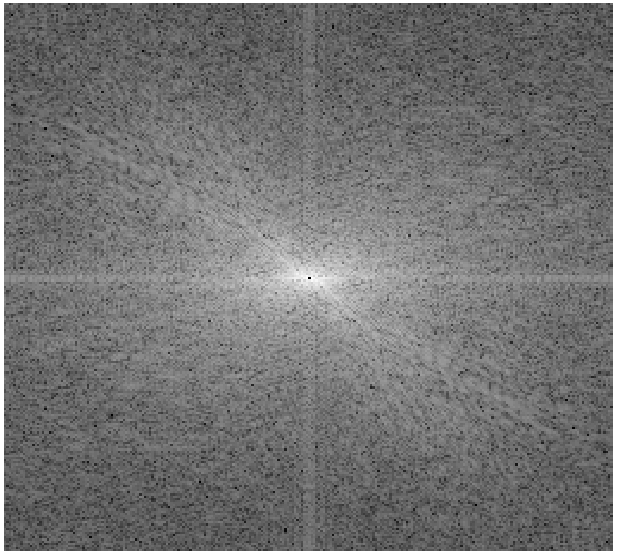


- **Limitations:**
  - cannot produce lines with one pixel width in all possible directions;
  - cannot distinguish between two blur kernels with nearby angles.

Natural Image Models

$f(x, y)$

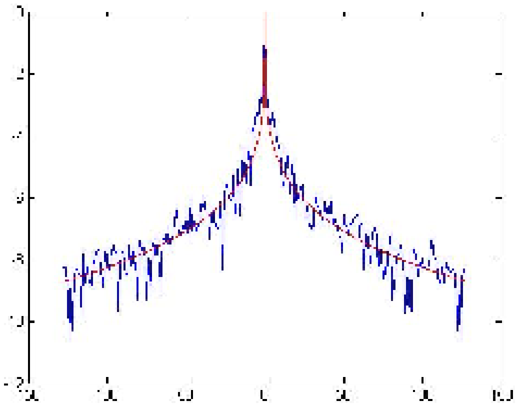
$\mathcal{FT}$



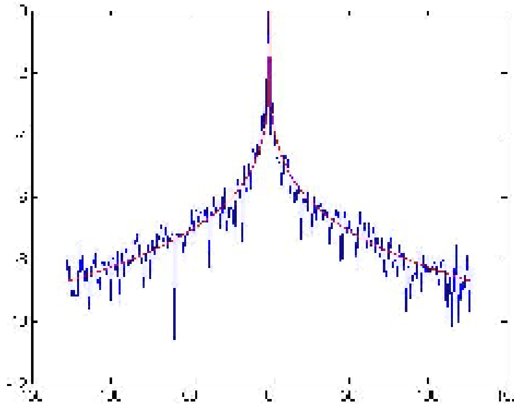
$\log|F(\xi, \eta)|$

- Global behavior of $\log|F(\xi, \eta)|$ along lines $\eta = \xi \tan \theta$ is monotonically decreasing with $|\xi|$. [Carasso, 2001]
- Approximate model: $\log|F(\xi, \xi \tan \theta)| \simeq -a|\xi|^b$, $a, b > 0$

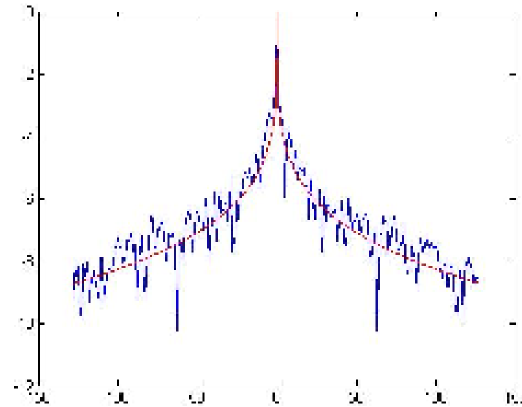
Natural Image Models



$\theta = 30^\circ$



$\theta = 45^\circ$



$\theta = 60^\circ$

$$\log |F(\xi, \xi \tan\theta)| \simeq -a\,|\xi|^b \qquad a = 3,\, b = 0.22$$

- Althought a, b can vary for different images, global behavior is approximately true regardless of the considered angle.

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Natural Image Models

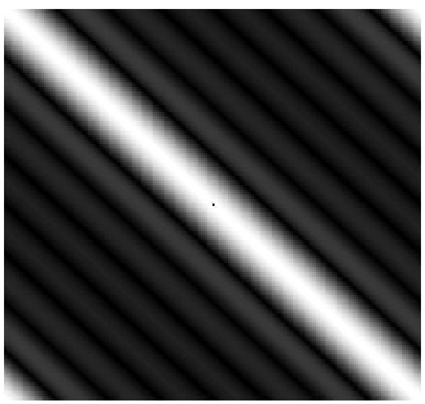
- Problem formulation in the Fourier domain:

$$G(\xi,\eta) = F(\xi,\eta)H(\xi,\eta) + W(\xi,\eta)$$

- Weak noise assumption:

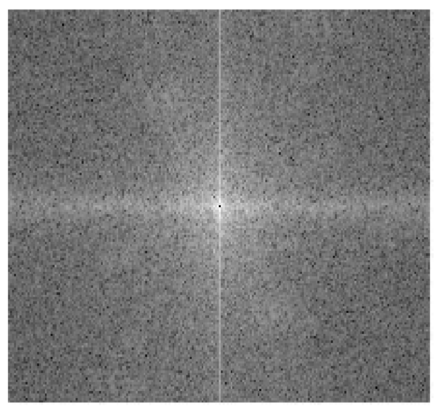
$$\log|G(\xi,\eta)| \approx \log|F(\xi,\eta)H(\xi,\eta)|$$

- Coarse behavior of $G(\xi,\eta)$ depends essentially on $F(\xi,\eta)H(\xi,\eta)$
- The sinc-like structure of $H(\xi,\eta)$ is preserved in $G(\xi,\eta)$, ie, the zeros of this sinc function become local minima.



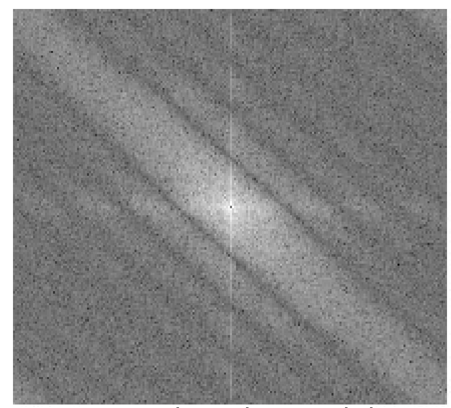
$|H(\xi,\eta)|$
($L = 8,\ \theta = 45^\circ$)

$\times$



$\log|F(\xi,\eta)|$
(Goldhill)

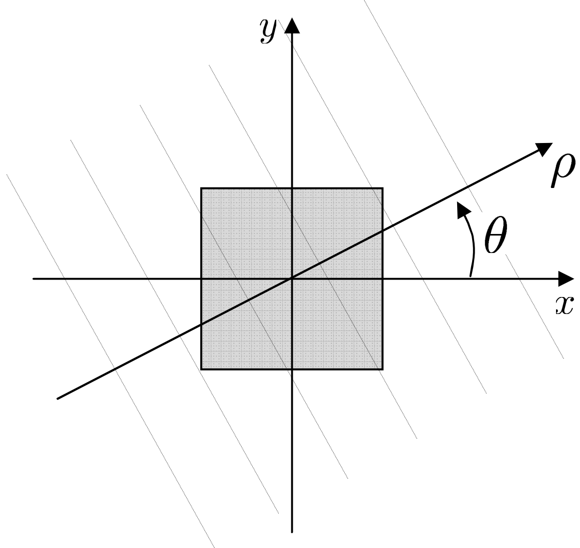
$\approx$

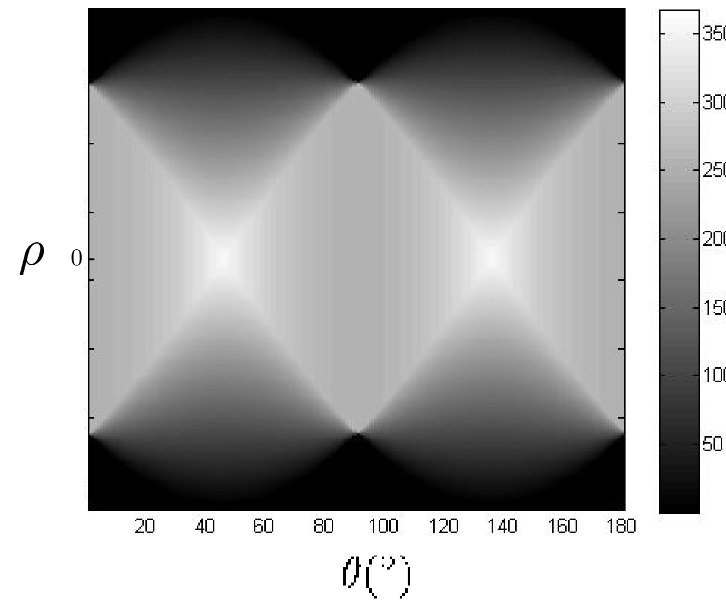


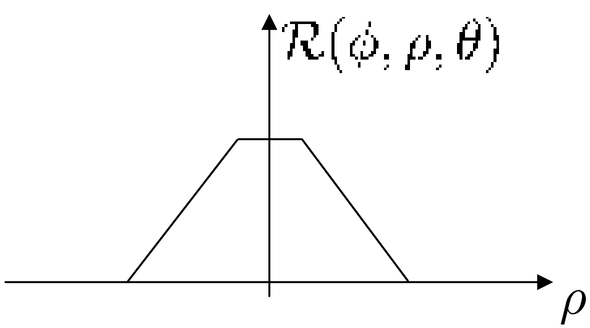
$\log|G(\xi,\eta)|$
(BSNR=30dB)

The Radon Transform

- Definition: $$\mathcal{R}(\phi,\rho,\theta)=\int_{-\infty}^{\infty}\phi(\rho\cos\theta-s\sin\theta,\rho\sin\theta+s\cos\theta)\,ds$$



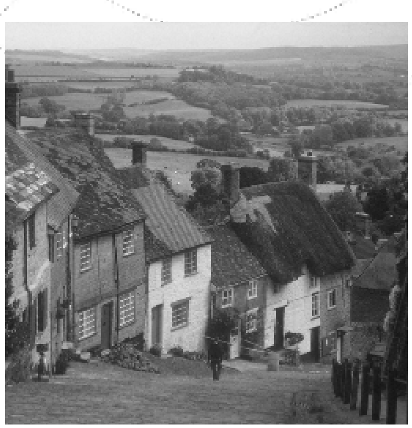




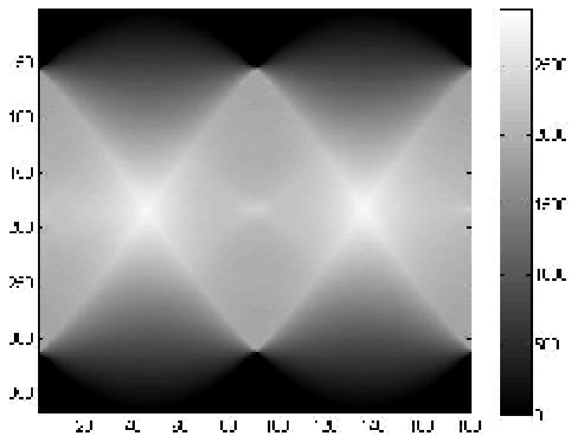
Angle estimation

- RT obtained by integrating on similar intervals, *i.e.*, with same area for any direction, will also be approximately equal:

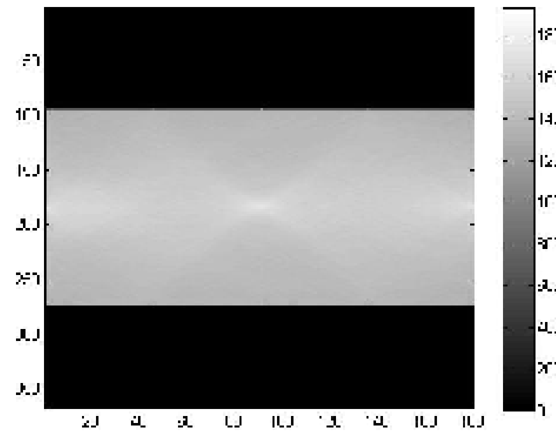
$$\mathcal{R}_d(f,\rho,\theta) = \int_{-d}^{d} f(\rho\cos\theta - s\sin\theta, \rho\sin\theta + s\cos\theta)ds,\ d = \frac{N}{\sqrt{2}};\ \rho \in [-d, d]$$



$f(x,y)$



$\mathcal{R}(\log|F(\xi,\eta)|,\rho,\theta)$



$\mathcal{R}_d(\log|F(\xi,\eta)|,\rho,\theta)$

- This RT of $\log|F(\xi,\eta)|$ has approximately the same variance, independently of θ .

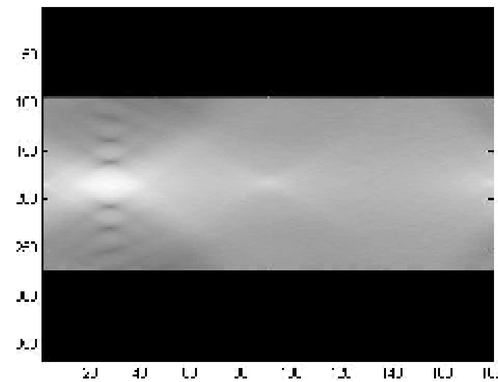
Angle estimation

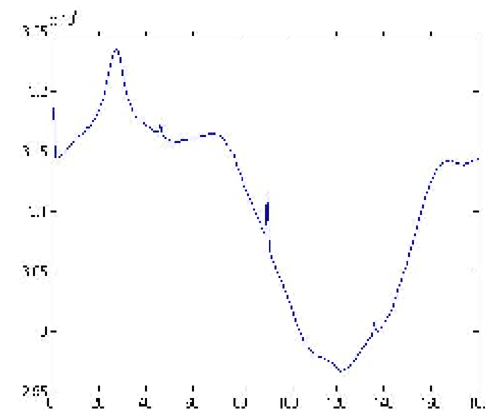
- Thus, a reasonable estimate of θ is

$$\boxed{\widehat{\theta} = \arg\max_{\theta} \, \mathrm{var}_{\rho} \left\{ \mathcal{R}_d(\log|G(\xi,\eta)|, \rho, \theta) \right\}}$$

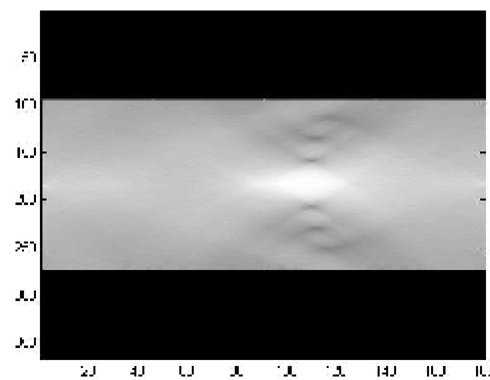
- Examples:

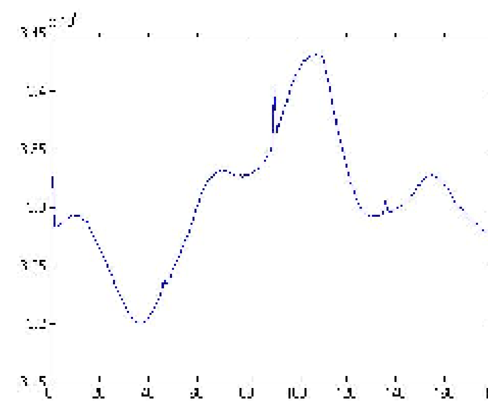
$L = 10$
$\theta = 30^{\circ}$





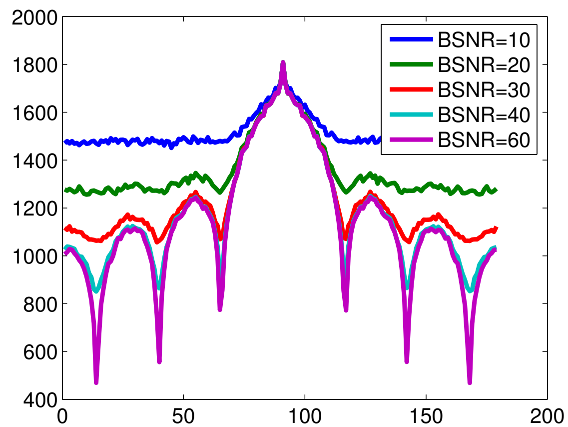
$L = 10$
$\theta = 110^{\circ}$

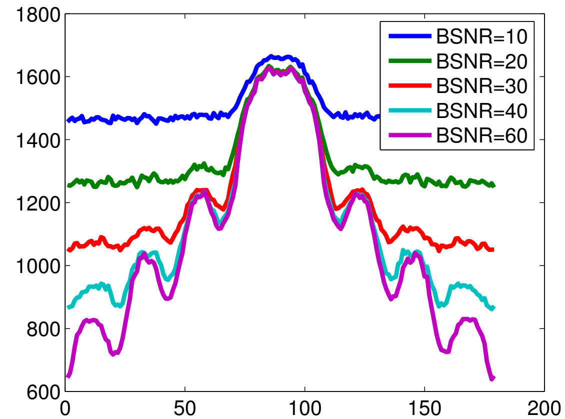




Length estimation

- Let the RT at the estimated angle $\widehat{\theta}$ be written as
  $\Pi(\omega) = \mathcal{R}_d(\log|G(\xi,\eta)|, \omega, \widehat{\theta})$





- Consider that $\Pi(\omega)$ is indeed the Fourier transform of a rectangular pulse of length L_S :

$$\Pi(\omega) = e^{j\psi(\omega)} \frac{\sin(\frac{\omega L_S}{2})}{\sin(\frac{\omega}{2})}$$

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Length estimation

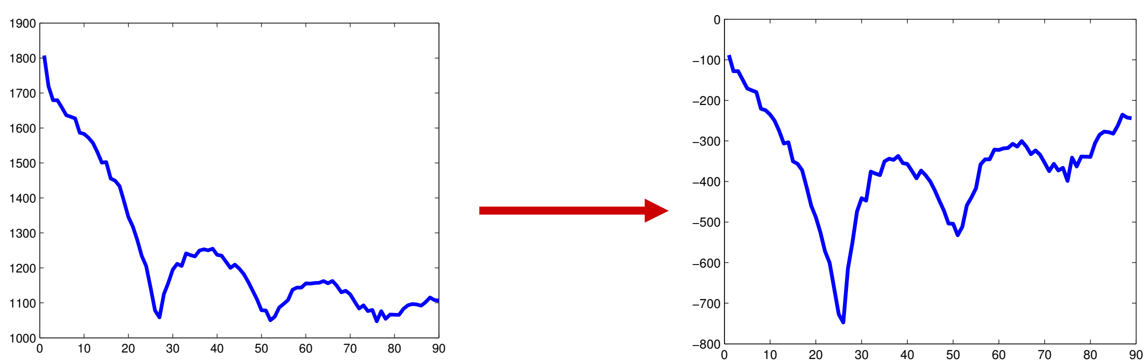
- Our goal is to find L_S by seeking for the first positive zero of $\Pi(\omega)$ given by $\omega_0 = 2\pi / L_S$.
- Since $\mathcal{R}_u$ was obtained based on an M point FFT,

$$\omega_0 = \frac{2\pi}{L_S} \approx \frac{2\pi}{M} i$$

- Thus:

$$\boxed{\widehat{L}_S \quad \mathrm{round}(M/i)}$$

- L_S is obtained via an heuristic algorithm. Consider the “transform”



Length Estimation

Algorithm 1 Length Estimation Algorithm

1: Compute the differences $\Delta_i = H(\omega_i) - H(\omega_{i-1})$ (only for $\omega > 0$)

2: Compute

$$\Delta_i^* = \begin{cases} p\,\Delta_i, & if\ \Delta_i > 0 \\ \Delta_i, & otherwise \end{cases} \tag{1}$$

where $p = 3$.

3: Compute the cumulative sums $\{S_1, S_2, ...\}$ where $S_i = \sum_{j=1}^{i} \Delta_i^*$.

4: Find the minimum $\hat{L}_S = \min\{S_1, S_2, ...\}$.

5: Compute

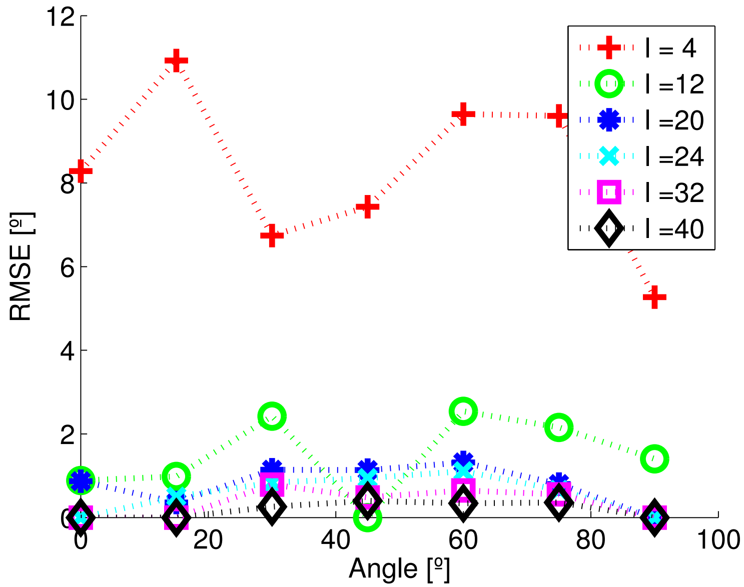
$$\hat{L} = \frac{N}{\hat{L}_S} C \tag{2}$$

where C is correction term is given by

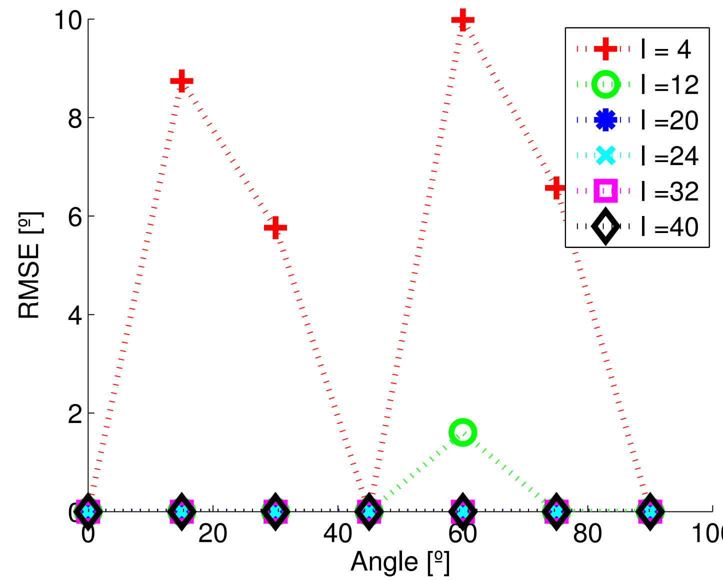
$$C = \begin{cases} \cos(\hat{\theta}), & if\ |\hat{\theta}| \leq \pi/4 \\ \sin(\hat{\theta}), & if\ \pi/4 < |\hat{\theta}| \leq 3\pi/4 \end{cases} \tag{3}$$

Angle estimation (Results)

- Given a blur length L , let $\theta_m(L)$ be the middle interval that leads to the same blur kernel.
- The error is given by: $\text{error} = \hat{\theta}_m(L) - \theta_m(L)$



BSNR = 10dB



BSNR = 40dB

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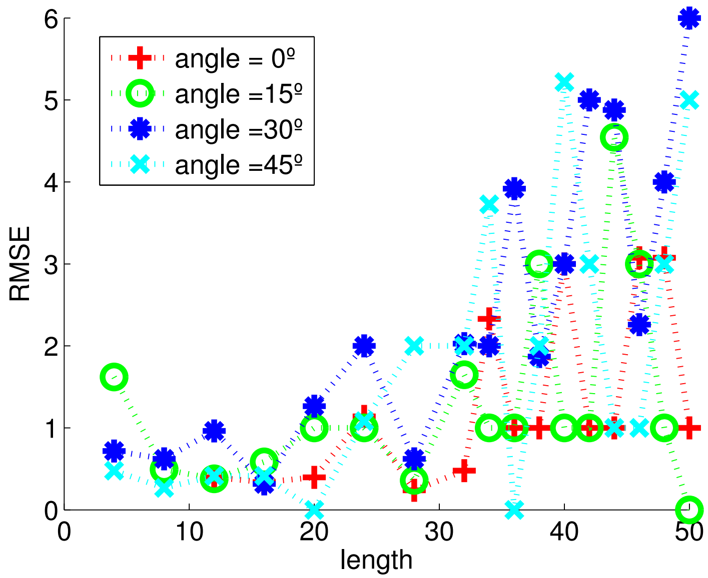
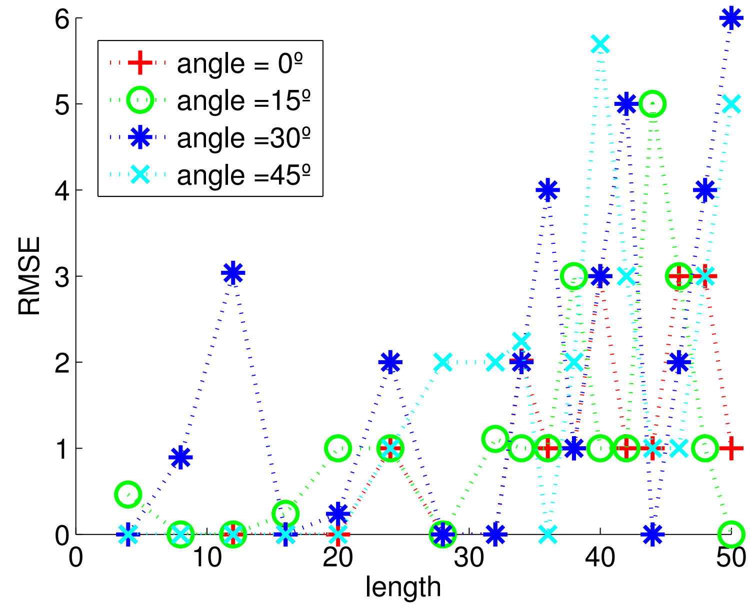
Length estimation (Results)



BSNR = 15dB



BSNR = 40dB

Conclusions and future work

- A robust algorithm was introduced to infer motion blur parameters.
- Quasi rotation invariance of the spectrum of natural images was exploited in the Radon domain.
- Address the reconstruction problem: how to handle small errors on the PSF?
- Model the PSF more realistically (for $\theta \notin \{0^\circ, 90^\circ\}$)
- Improve length estimation.