Assumption 1: Dirichlet distribution (alternative justification)

- **Let a1=N_Ut+1, a2=N_Vt+1 and a3=N_Rt+1 be the parameters of a Dirichlet distribution**

```
a1 = 2;
a2 = 1;
a3 = 501;
a0 = a1 + a2 + a3;
```

- **Let exp be the density of the (Ut,Vt)~Dirichlet(a1,a2,a3) up to normalization in the 2-simplex**

```
exp = u ^ (a1 - 1) v^ (a2 - 1) (1 - v - u) ^ (a3 - 1);
```

- **The normalization constant M (cross-checked analytically)**

```
M = NIntegrate[exp, {u, 0, 1}, {v, 0, 1 - u}]
```

7.90479×10^{-9}

```
N[Gamma[a1] * Gamma[a2] * Gamma[a3] / (Gamma[a0])]
```

7.90479×10^{-9}

- **Let ea1 be the expected value of Ut and sd its standard deviation**

```
ea1 = a1 / a0;
sd = Sqrt[ea1 (1 - ea1) / (a0 + 1)];
```

- **Take, for instance, p as...**

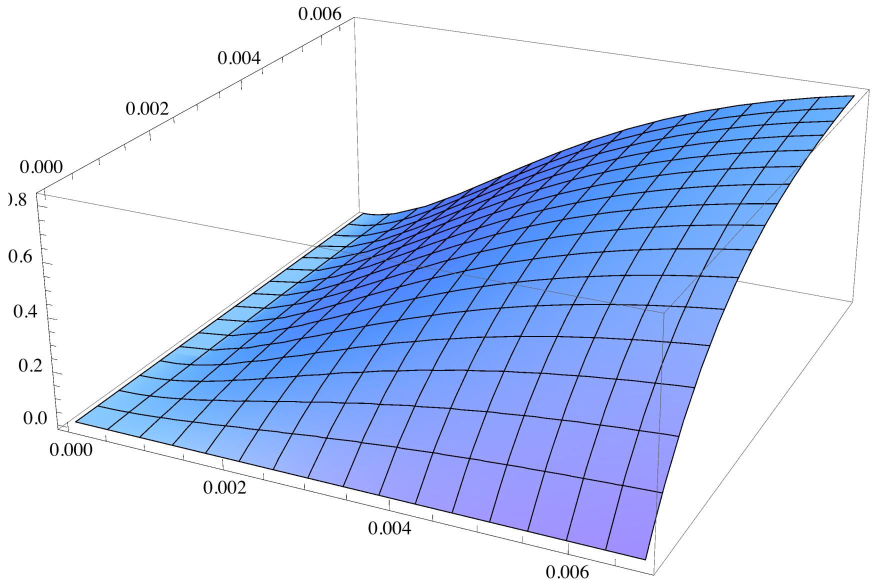
```
p = ea1 + sd;
```

```
N[p]
```

0.00676589

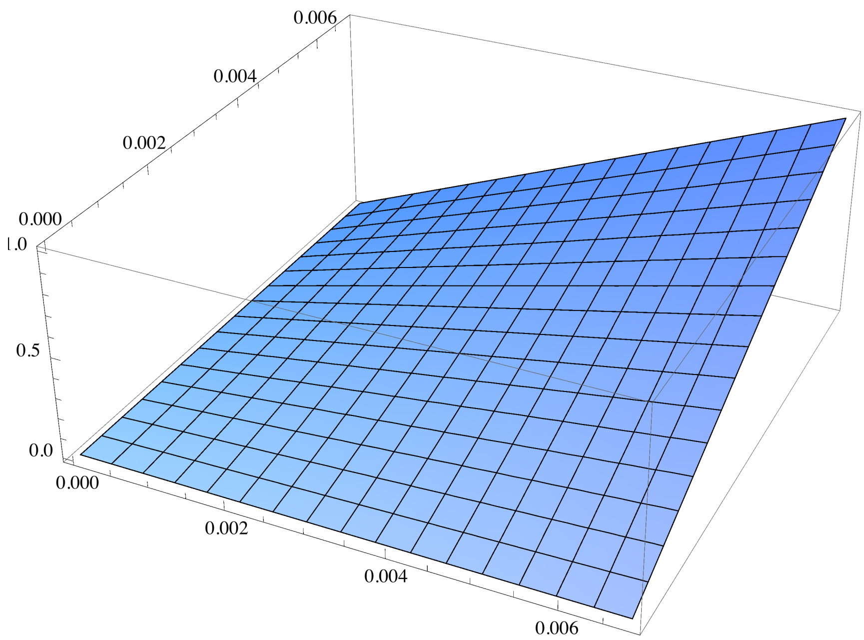
- **The cumulative distribution function of the Dirichlet(a1,a2,a3) in the square [0, p] x[0, p]**

```
G1 = Plot3D[
  NIntegrate[exp / M, {u, 0, x}, {v, 0, y}], {x, 0, p}, {y, 0, p}]
```



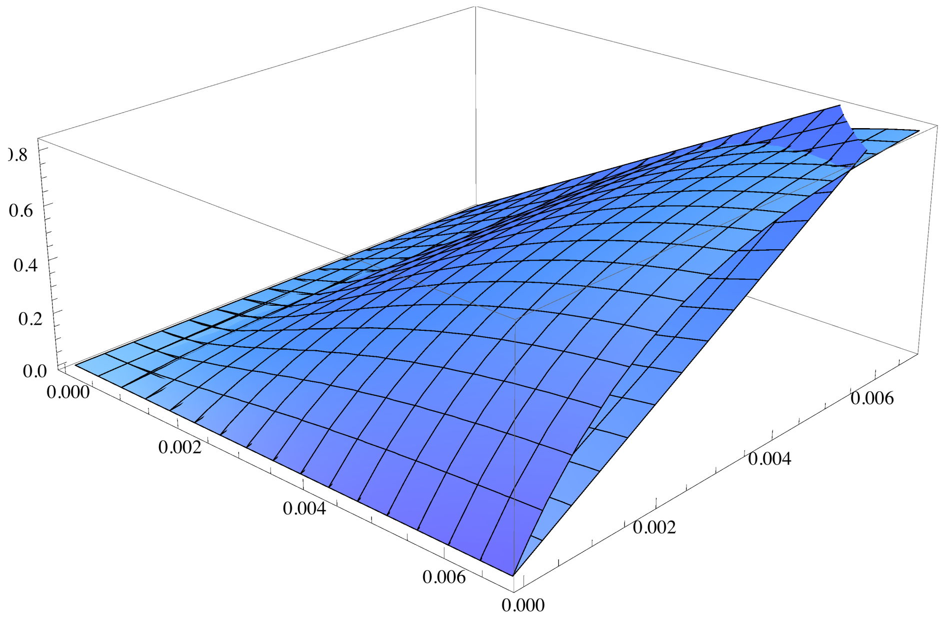
- **The cumulative distribution function of the Uniform([0,p]x[0,p]) in the square [0,p]x[0,p]**

```
G2 = Plot3D[NIntegrate[1 / p^2, {u, 0, x}, {v, 0, y}], {x, 0, p}, {y, 0, p}]
```



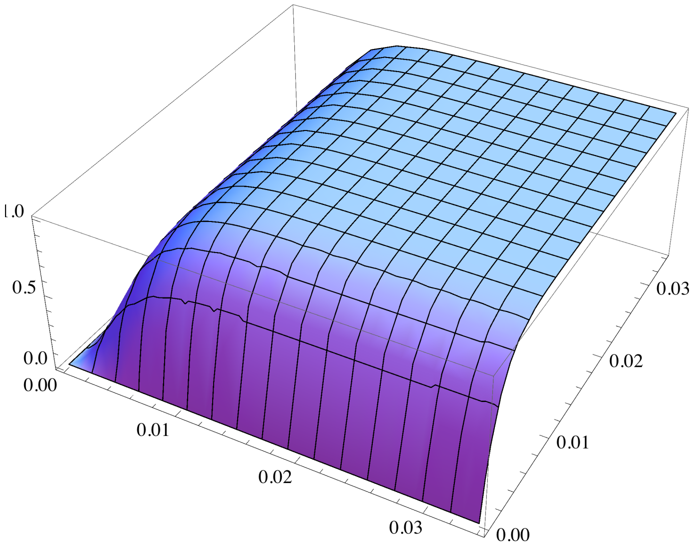
- **Comparing one with the other in the square [0,p]x[0,p]**

```
Show[G1, G2]
```



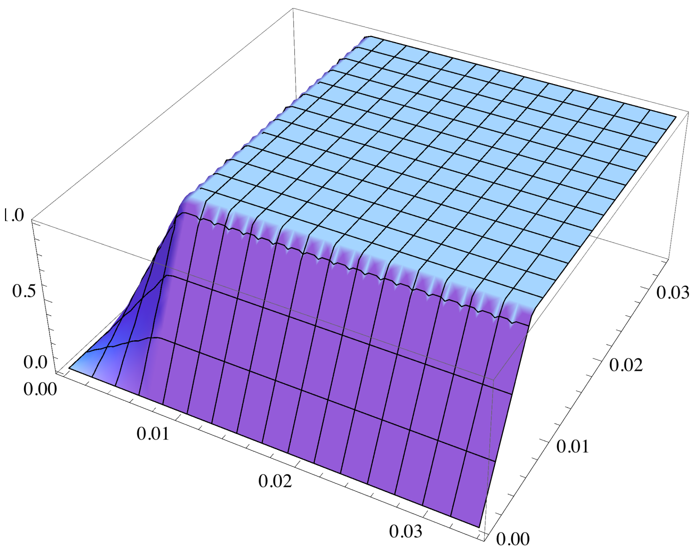
- **The cumulative distribution function of the Dirichlet(a1,a2,a3) in the square [0, 5p] x[0, 5p]**

```
G1 = Plot3D[
  NIntegrate[exp / M, {u, 0, x}, {v, 0, y}], {x, 0, 5 * p}, {y, 0, 5 * p}]
```



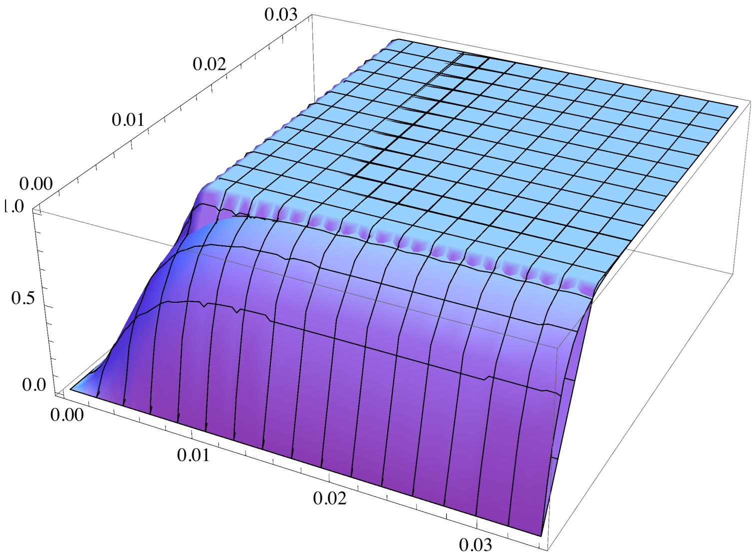
- **The cumulative distribution function of the Uniform([0,p]x[0,p]) in the square [0, 5p] x[0, 5p]**

```
G2 = Plot3D[NIntegrate[If[u ≤ p && v ≤ p, N[1 / p^2], 0.], {u, 0, x}, {v, 0, y}],
  {x, 0, 5 * p}, {y, 0, 5 * p}]
```



■ Comparing one with the other in the square [0,5p]x[0,5p]

```
Show[G1, G2]
```




```
Show[G1, G2]
```